MULTI USER CHAOTIC COMMUNICATION SYSTEMS USING ORTHOGONAL CHAOTIC VECTORS

A thesis submitted in partial fulfilment of the requirements for the degree of

Master of Technology (Research)
in
Electronics & Communication Engineering

By

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CERTIFICATE

This is to certify that the thesis titled "**Multi User Chaotic Communication Systems Using Orthogonal Chaotic Vectors**" submitted to the National Institute of Technology, Rourkela by **Venkatesh S**, Roll No. **609EC102** for the award of the degree of **Master of Technology (Research)** in Electronics & Communication Engineering, is a bonafide record of research work carried out by him under my supervision and guidance.

The candidate has fulfilled all the prescribed requirements.

The thesis, which is based on candidate's own work, has not been submitted elsewhere for a degree/diploma.

In my opinion, the thesis is of standard required for the award of a Master of Technology (Research) degree in Electronics & Communication Engineering.

To the best of my knowledge, he bears a good moral character and decent behaviour.

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Venkatesh S

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List of Acronyms

Acronym	Reference
ACSK	Antipodal Chaos Shift Keying
AWGN	Additive White Gaussian Noise
CSE	Chaotic Sequence Estimator
CSK	Chaos Shift Keying
DCSK	Differential Chaos Shift Keying
GCDSK	Generalized Correlation Delay Shift Keying
IA	Inverse Averaging
LMS	Least Mean Square
MAI	Multiple Access Interference
MMSE	Minimum Mean Square Error
MUI	Multi User Interference
NLMS	Normalized Least Mean Square
OCV	Orthogonal Chaotic Vector
RLS	Recursive Least Square

Abstract

Due to quasi orthogonal nature of chaotic spreading sequences, the co-channel interference will be introduced and increases with the increase in number of users and limits the number of simultaneous users transmitting information. In this dissertation the application of orthogonal chaotic vector (OCV) for spreading information bits is presented. The bit error rate of the multi user chaotic communication system is analysed through simulation and analytical expressions. Two main types of communication scenarios are considered, multi user chaotic communication system with coherent receiver and training assisted non coherent multi user chaotic communication system with adaptive receiver. The first case deals with ideal scenario where it is assumed that exact replica of chaotic vector used to spread data is available at the receiver and the information is extracted without synchronisation error. Thus lacks practical realisation but, the results obtained will provide a lower bound for comparison with other practical counter parts. The second scenario deals with more practical approach where a reference chaotic sequence is also transmitted by modulating with training bits so that the chaotic vector required for correlation can be recovered at the receiver.

1. Introduction

1.1 What is Chaos?

Chaos is an aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions [1].

The three components of the definition are clarified as follows:

1. Random nature or noise like appearance [1,2,3]: The chaotic sequences generated from chaotic systems will have noise like appearance (Fig. 1.1). Theoretically, if the chaotic system is operated in chaotic region, then the chaotic system oscillates with infinite orbits with no value repeated twice [2].

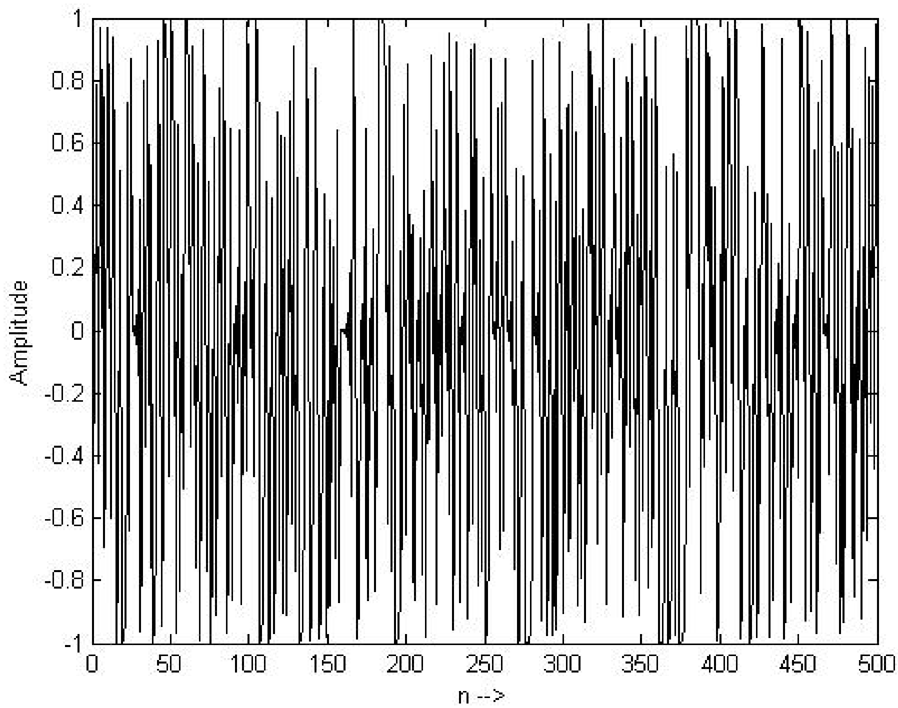


Fig. 1.1 First 500 samples of Chaotic sequence generated from cubic map

2. "Deterministic" [1,2,3] systems can have no stochastic (meaning probabilistic) parameters. It is a common misconception that chaotic systems are noisy systems driven by random processes. The irregular behaviour of chaotic systems arises from intrinsic nonlinearities rather than noise.

3. Sensitive to small variations in initial condition [1,2,3]: Which means the chaotic sequences generated from the same chaotic system but with different initial condition with very small difference can also make a huge difference in outcome of the chaotic system in future iterations. In Fig. 1.2 the chaotic sequences generated from cubic

map is plotted for two different values of initial condition (0.1 and 0.1 + 1e-6). The sensitivity of cubic map for initial condition is clearly evident in Fig. 1.2.

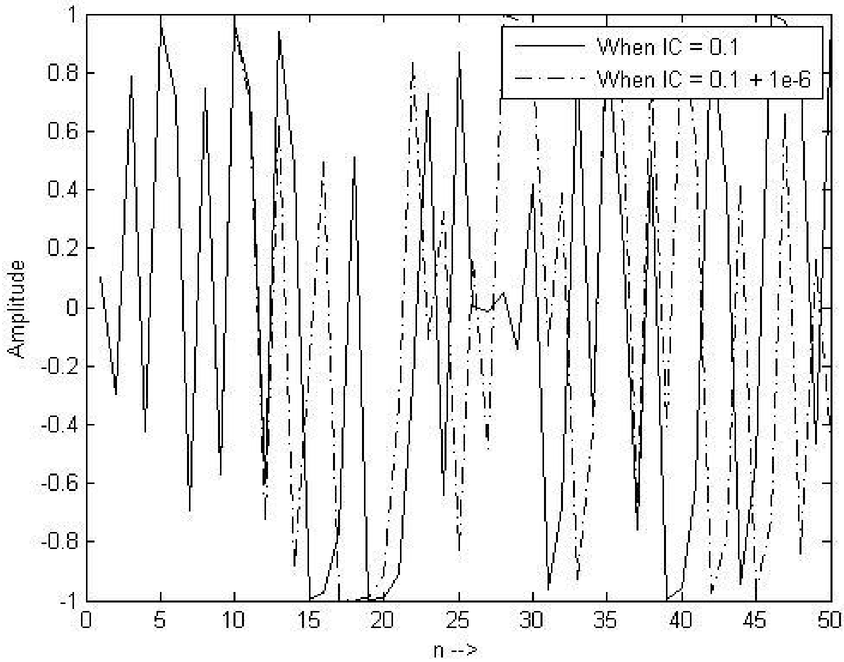


Fig. 1.2 Chaotic sequence generated from cubic map for different initial conditions

1.2 Why Chaos for Communication?

In [4] Shannon, showed that the channel capacity of the communication link is optimized when maximum entropy signal is used for transmission. Chaotic signals are the class of the signals which inherits the properties required for maximum entropy signal. Chaotic communication systems can be viewed as generalized direct sequence spread spectrum systems. Chaotic communication systems inherits all the advantages that a conventional spread spectrum has like, robust against multipath impairments, immune to narrow band interference and multi user capability. A chaotic sequence or system evolves in a seemingly random fashion, while the direct sequence system is limited to a small finite set of values.

1.2.1 Chaotic Communication Systems

In [4] Shannon stated that, an ideal system is the one which can transmit data at rate 'C' without error.

$$C = W \log_2 \left(1 + \frac{P}{N_o} \right)$$

Where, W is the channel bandwidth, P is the signal power and N_o is the noise power. In [4] Shannon also stated that the channel capacity of the communication system will approach to that of ideal system when the statistical properties of the transmitting signal is same as that of white noise (Impulse autocorrelation and flat wide band PSD). Shannon expanded these results to communications through a noisy channel [5] as well as capacity measures for secure communications [6].

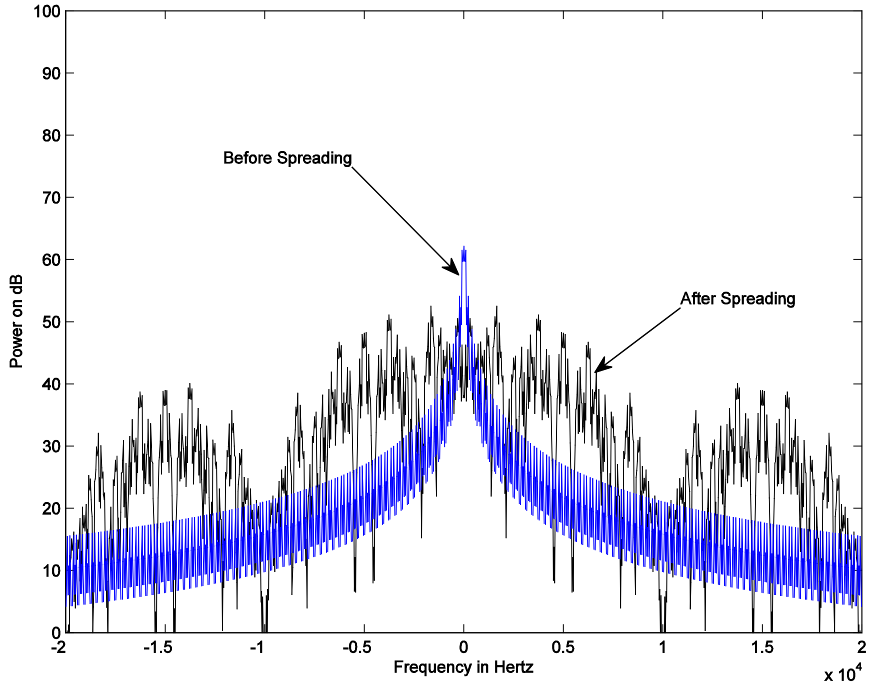


Fig. 1.3 Power spectral density effects of signal spreading.

Spread-spectrum signals are well known to be resistant to interferers (natural and man- made) and multipath effects, conducive to secure communications by lowering the average spectral density, and effective for use in multiple access systems where users simultaneously re-use the shared communications bandwidth [7]. In Fig. 1.3 the spectrum of the information signal before and after spreading is shown. The extension of spread-spectrum signalling techniques to chaotic communications gained active interest in the early 1990s [8,9,10,11] since frequency bandlimited chaotic spreading sequences are known to closely mimic Shannon's ideal noise-like waveform [4]; the chaotic wave- form is a near-optimal approximation of a transmission with maximum capacity for carrying information in a Gaussian white noise channel. Compared to other spread communication systems, chaotic waveforms may be viewed as having the potential for higher throughputs (as a result of higher SNR) or a lower power spectral density (increasing spectral re-use) for the same data throughput. Further, the impulsive autocorrelation also gives chaotic waveforms superior multipath and co-interference characteristics as compared to traditional spread-spectrum signals like CDMA.

By contrast with a conventional digital modulation scheme, where the transmitted symbols are mapped to a finite set of periodic waveform segments for transmission, every transmitted symbol in a chaotic modulation scheme produces a different nonperiodic waveform segment. Be- cause the cross correlations between pieces of periodic segments are lower than between pieces of periodic waveforms, chaotic modulation ought to offer better performance under multipath propagation conditions. Thus, chaotic modulation offers a potentially simple solution for robust wideband communications. [12]

The fundamental difference between a traditional direct sequence spread-spectrum communication system and a coherent chaotic sequence spread spectrum communication system is the absence of apparent periodicity in the chaotic waveform.

Table 1.1 Major differences between conventional spread spectrum communication system and chaotic communication system

	Conventional Spread Spectrum Communication	**Chaotic Communication**
Spreading Sequence	Pseudo Random Binary Sequence	Sequence of Real numbers with interval (-1,1)
Generation of Spreading Sequence	Using linear feedback shift register	Deterministic Discrete dynamical system
Length of the spreading sequence	Depends on the length of shift register or the degree of the polynomial	Depends on the precision of the system used to generate chaotic sequence.

1.2.2 Generation of Chaotic Sequence and Descriptors of chaos

A chaotic system is defined by three fundamental properties [2]:

1. A chaotic system has a dense collection of points with periodic orbits
2. A chaotic system is extremely sensitive to initial conditions and perturbations
3. A chaotic system is topologically transitive

Mathematically the systems with above mentioned characteristics are described using, nonlinear differential and difference equations. Notionally, a chaotic system operates on some open continuous domain and takes every possible value within an open continuous range (denseness); is periodic with potentially non-measurable period; responds in a divergent manner to the smallest of perturbations (extreme sensitivity);

and recursively operates on any value in the domain to produce a value that is arbitrarily close to any chosen value in the range (topological transitivity). The afore mentioned characteristics of the chaos are quantified using descriptors like, bifurcation diagram, lyapunov exponents and entropy of the chaotic sequences generated.

Lorenz attractor [13] is one of the most discussed chaotic system which is described using nonlinear differential equations:

$$\dot{x} = \sigma\left(y - x\right)$$

$$\dot{y} = rx - y - xz$$

$$\dot{z} = xy - bz$$

The system of equations defined above will be in chaotic regime when, σ = 16, b = 4, and r = 45.92. The solutions for the Lorenz attractor are plotted in Fig. 1.4 and 1.5 for initial conditions $x = y = z = 0.1$.

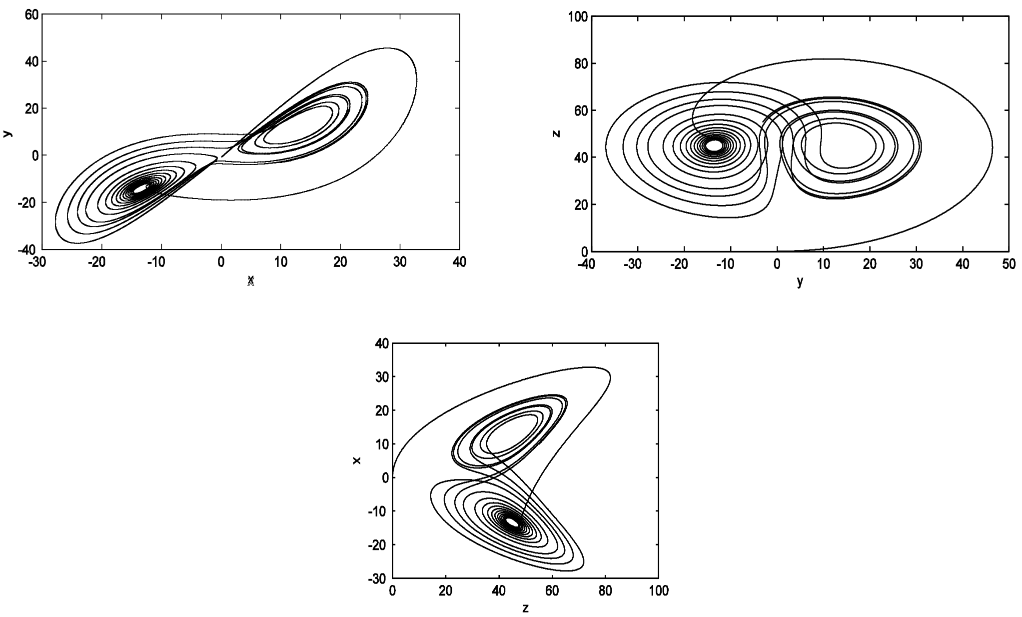


Fig. 1.4Solutions of Lorenz attractor: Projection on *x*- *y* plane(top left), *z*-*y* plane(top right) and *x*-*z* plane(bottom)

Numerous discrete-time mappings have been proven to exhibit all three properties of a chaotic system. Some of the difference equations which describe discrete chaotic systems are, Chebyshev map [14], and the logistic equation [15]. The difference equation representation of chebyshev and logistic map is :

Chebyshev map: $x_{n+1} = \cos\left(k\cos^{-1}\left(x_n\right)\right)$

Logistic map: $x_{n+1} = \mu x_n\left(1 - x_n\right)$

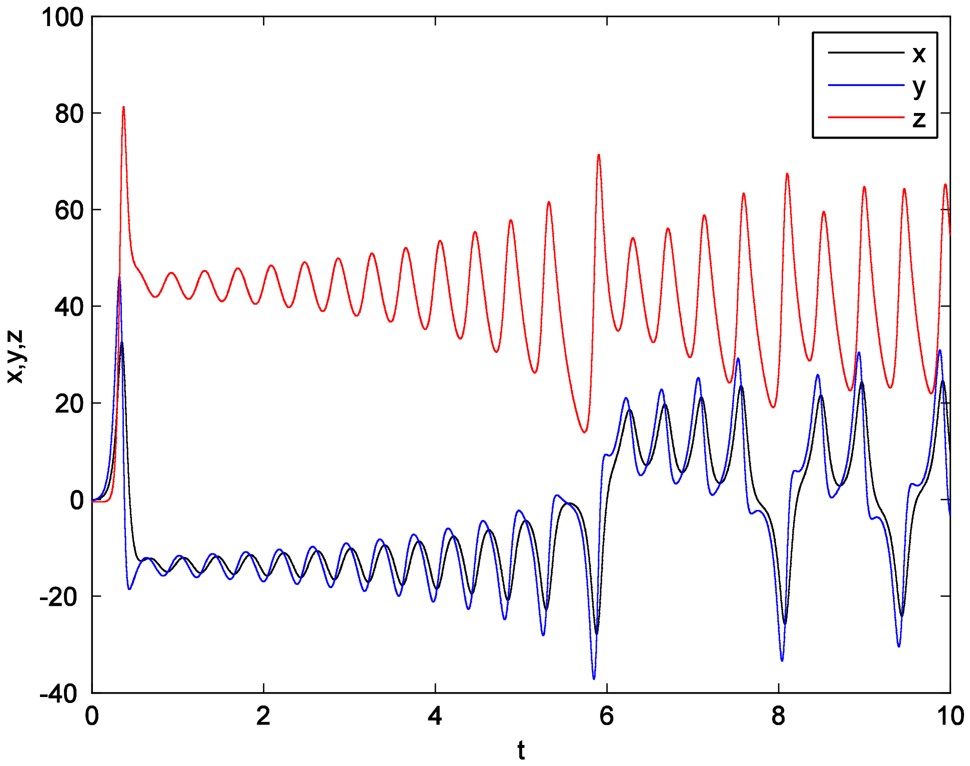


Fig. 1.5Solutions of Lorenz attractor for σ = 16, b = 4 and r = 45.92 and initial conditions x = y = z = =0.1

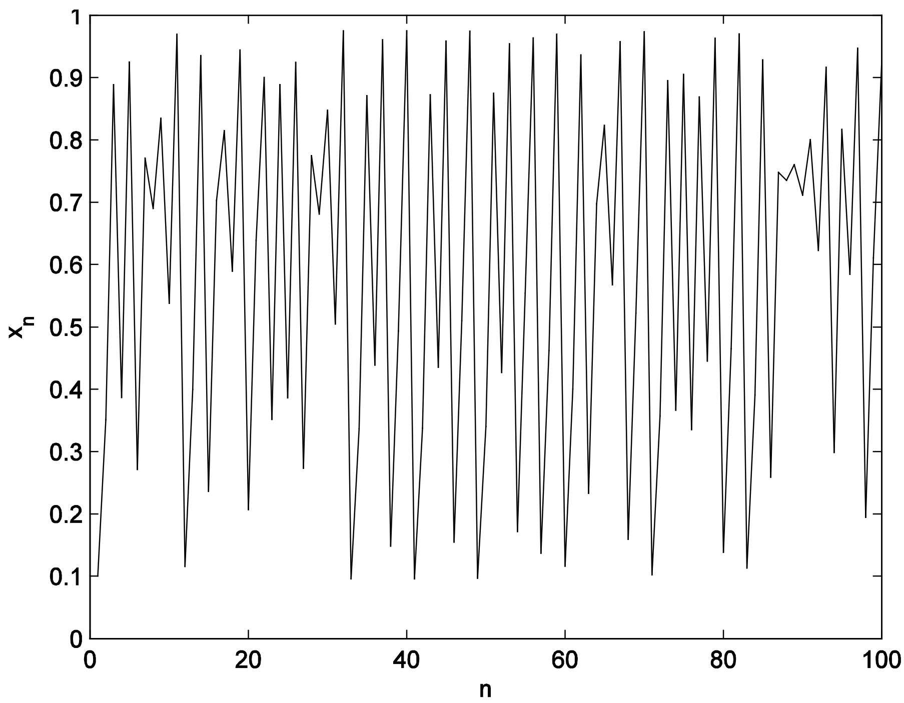


Fig. 1.6 First 100 samples generated using logistic map for $\mu = 3.9$ and initial condition $x_1 = 0.1$

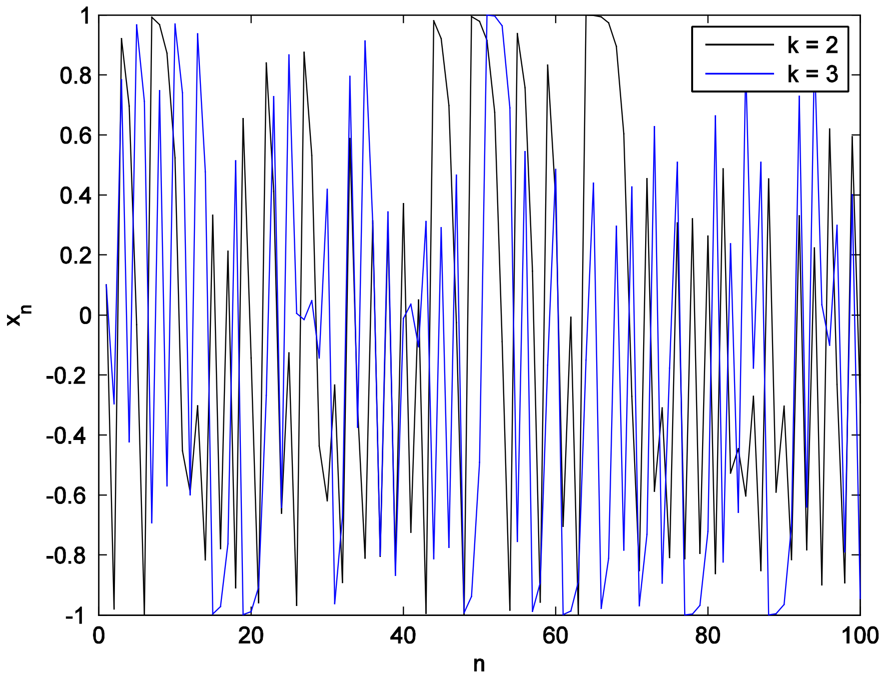


Fig. 1.7 First 100 samples of Chebyshev map of order 2 and 3 for initial condition 0.1

A. Descriptors of chaos:

Some of the descriptors which are used to quantify the chaoticity of a chaotic system are:

Bifurcation diagram:

An important feature of chaotic sequence is period doubling. The bifurcation where the period doubling occurs is called *pitchfork bifurcation*. Bifurcation diagram is used to analyse the sudden changes in the behavior of chaotic systems with respect to the changes in parameter values.

Bifurcation diagrams are generated by iterating chaotic map for parameter values increased in small steps and plotting the orbits obtained for particular parameter value versus parameter value. In Fig. 1.8 and Fig. 1.9 the bifurcation diagram for logistic map for $2.6 \le \mu \le 4$ and chebyshey map for $0.5 \le k \le 3$ is shown. From bifurcation diagrams we can observe that the first period doubling occurs at $\mu \ge 3.1$ for logistic map and at $k = 1$ for chebyshev map. After further period doubling the logistic map and chebyshev map enters in to chaotic regime for $\mu \ge 3.5699$ and for $k \ge 2$ respectively. For logistic map, in Fig. 1.8, the void space in between the dense collection of points for $\mu > 3.83$ and $\mu < 3.84$ is called as saddle point.

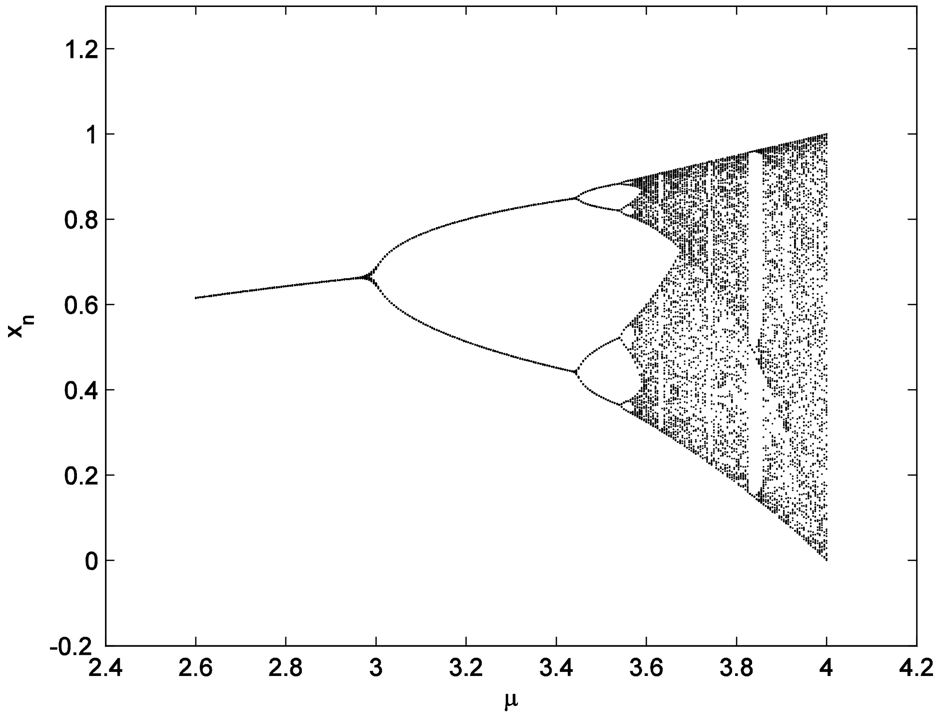


Fig. 1.8 Bifurcation diagram for logistic map

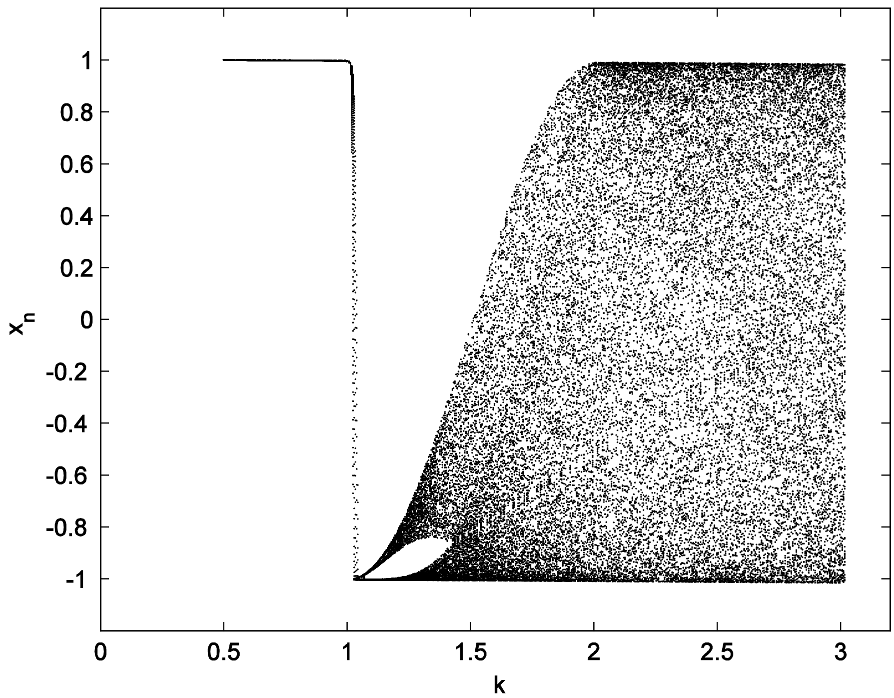


Fig. 1.9 Bifurcation diagram for Chebyshev map for 0.5<k<3

Lyapunov Exponent:

Lyapunov exponent is used to analyse the exponential divergence or convergence of nearby characteristics. The lyapunov exponent is computed using equation

$$\lambda = \frac{1}{n}\sum_{k=1}^{n}\ln\frac{d_k}{d_o}$$

Where d_o is the small separation between two initial conditions n is the number of iterations and $d_k = \left|x_{j+k} - x_{i+k}\right|$, $d_o = \left|x_j - x_i\right|$. The value of λ obtained using this equation is repeated for several trajectories and averaged to get average Lyapunov exponent for the attractor.

In Fig. 1.10 and Fig. 1.11 the Lyapunov exponents for logistic map and chebyshev maps are plotted. From the Lyapunov exponents obtained for logistic map and chebyshev map we can observe that, at the bifurcation points there will be a discontinuity. The values of Lyapunov exponent are negative when the chaotic map is in limit cycle region and are positive when the chaotic map is in chaotic regime. Thus from Fig. 1.10 we see that logistic map enters in to chaotic regime for μ > 3.56 and chebyshev map enters chaotic regime for k ≥ 2. The positive value of Lyapunov exponent indicates the exponential divergence of nearby trajectories and negative value indicates the convergence of nearby trajectories.

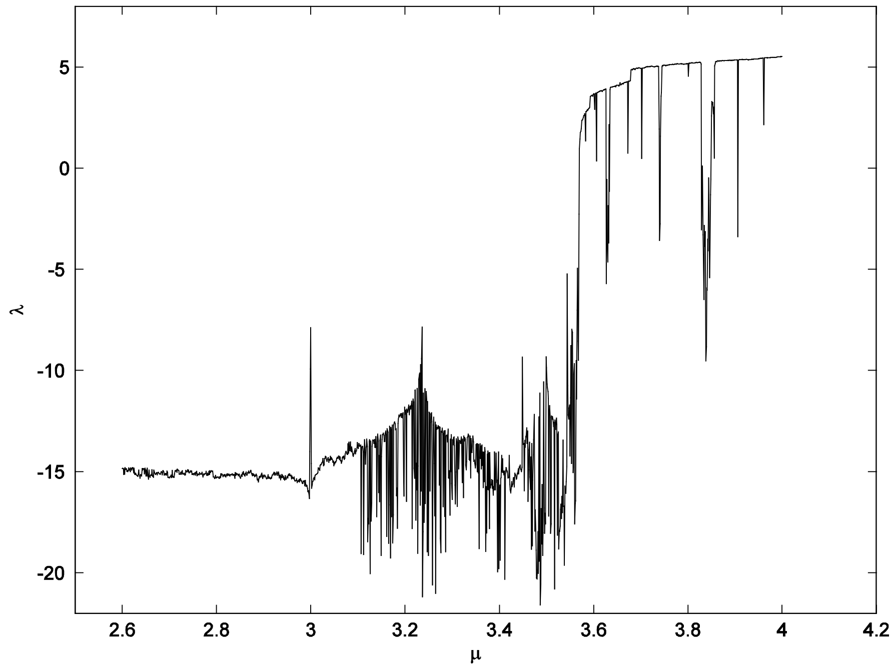


Fig. 1.10 Lyapunov exponent versus μ for logistic map

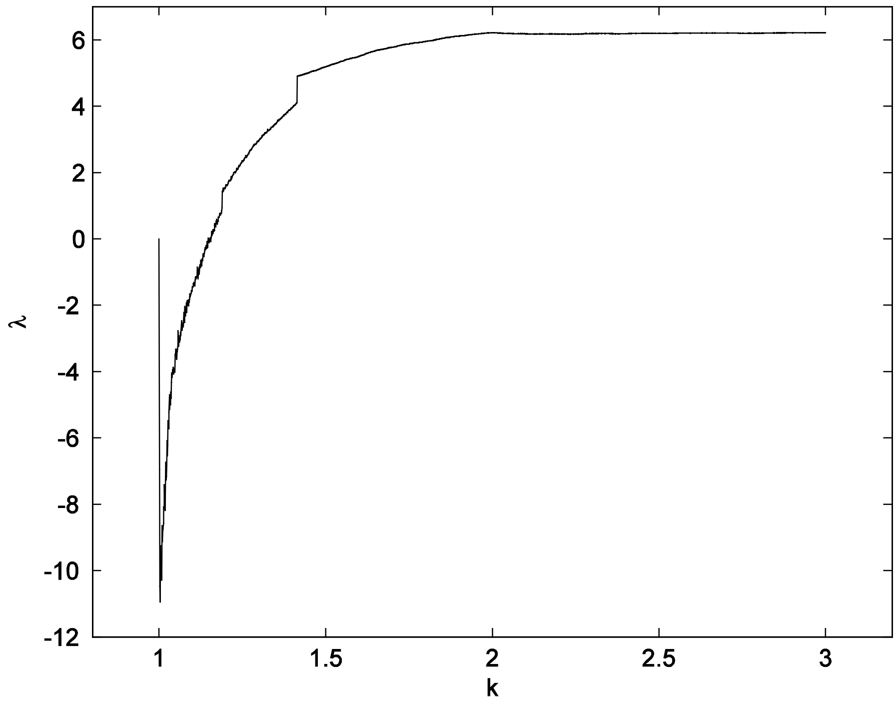


Fig. 1.11 Lyapunov Exponent versus ‘*k*’ for Chebyshev map

Entropy:

Entropy is the measure of randomness or the number of possible states for the chaotic system. The entropy of the chaotic system can be used to analyse the denseness of the chaotic regime. The entropy of the chaotic system is computed by first generating chaotic sequence of large length and then sampling the range of values into N bins to find the probabilities. After finding the probabilities the entropy is calculated using equation

$$S = -\sum_{k=1}^{N} p_i \log_2\left(p_i\right)$$

Where N is the number of bins and P_i is the probability of occurrence of i^{th} bin value in the chaotic sequence. The value of entropy S, reaches its maximum value when the probability of occurrence of all values are equal i.e., $P_i=(1/N)$, under this condition the maximum entropy will be

$$S = \log_2 N$$

In Fig. 1.12 and Fig. 1.13 the entropy for logistic map and chebyshev map are plotted. The entropies are computed by generating 1 million values and computing the probabilities for 10000 bins. From Fig. 1.12 we see that the entropy of the chaotic sequence generated using logistic map has maximum value for μ > 3.56 where as for chebyshev map it is k>2.

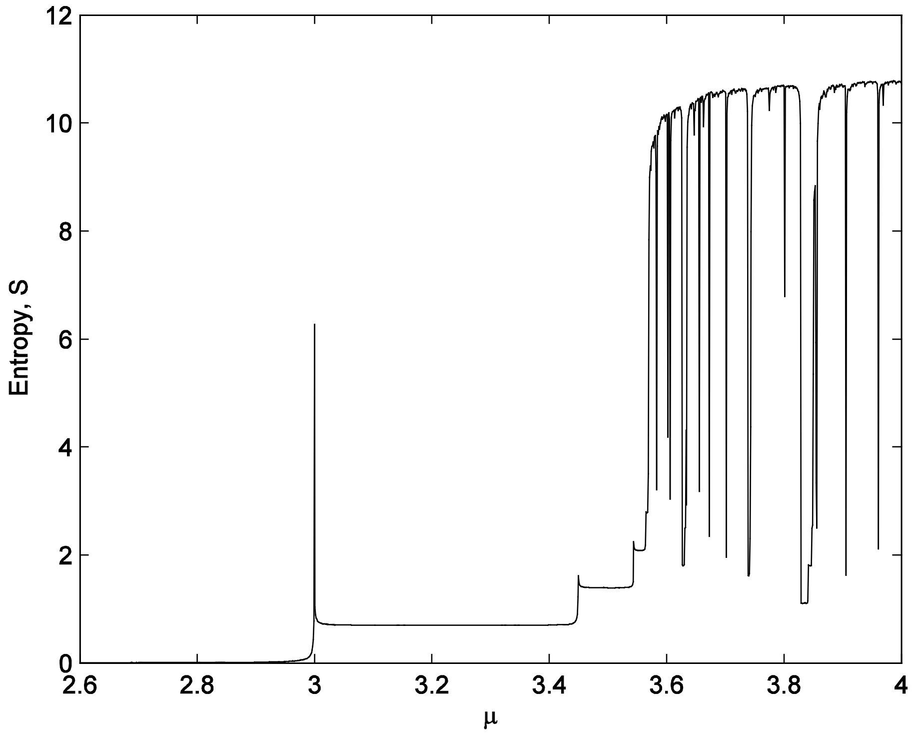


Fig 1.12 Entropy versus μ for logistic map

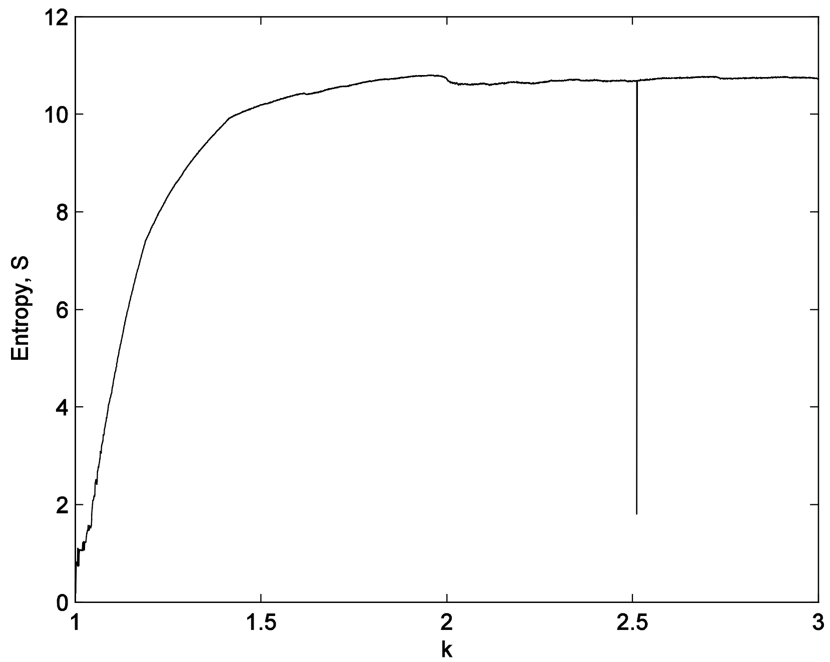


Fig. 1.13 Entropy versus '*k*' for Chebyshev map

1.3 Motivation

The characteristics of chaotic sequences such as, wideband nature (Fig. 1.14.), noise like appearance (Fig.1.1), low cross correlation and impulse like auto correlation functions (Fig. 1.15) [1,3] makes it to be an alternate solution for spread spectrum communication[3]. Due to the wideband nature of the chaotic sequences the chaotic communication system will be robust against multipath impairments than compared to the conventional narrowband communication system. Due to the sensitive dependence of the chaotic system on initial condition it is possible to generate the wide range of chaotic basis from the single chaotic system by just using different initial conditions without increasing the complexity of the hardware. The ability of the chaotic systems to generate wide range of chaotic sequences without costing for system hardware makes chaotic systems a better candidate for multiple access communication. A number of chaos-based communication schemes have been suggested, but many of these systems are very sensitive to distortion, filtering, and noise. However, a chaos-based communications system could also improve privacy, security, and probability of intercept, in as much as chaotic sequences, unlike pseudorandom sequences, can be made completely aperiodic.

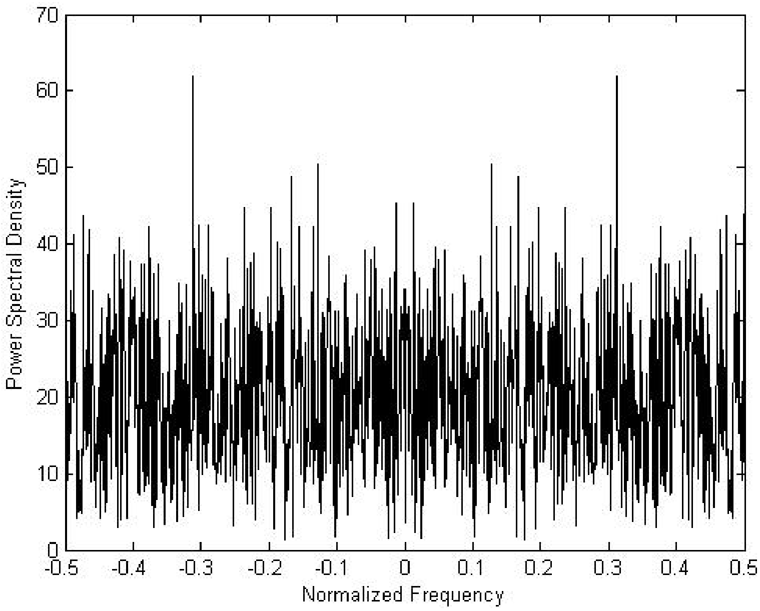


Fig. 1.14 Power Spectrum of the Chaotic sequence generated from cubic map

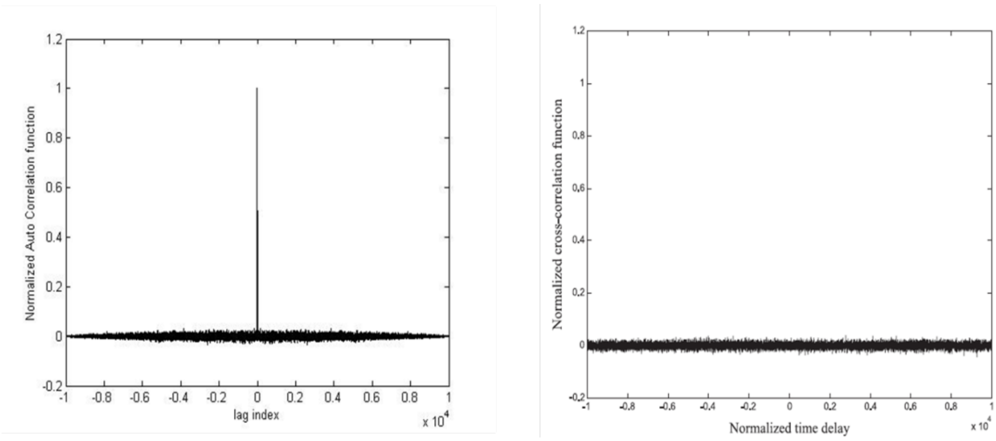


Fig. 1.15 Auto Correlation and Cross correlation function of chaotic functions

1. Quasi orthogonal characteristics [16,17,18,19]

Due to the quasi orthogonal characteristics of the chaotic signals there exist residual cross correlation coefficients under multiple access scenarios. This causes multi user interference and degrades bit error rate performance of the multi user chaotic communication system. However this effect will be negligible when spread factor i.e., number of chaotic samples used to spread the information bit is sufficiently large.

This problem arising from quasi orthogonal characteristics can be solved using orthogonal chaotic vectors. Since, the orthogonality of the orthogonal chaotic vectors doesn't depend on the spread factor and number of users.

2. Estimation problem due to variable bit energy [16,17,18]

The auto and cross correlation of the chaotic sequences evaluated for bit duration is a random variable with its mean value and variance which depends on the chaotic map used to generate the chaotic sequence. Less the value of variance of this random variable better the bit error rate performance.

The estimation problem arising from the random nature of auto and cross correlation products of the chaotic sequences can be solved by using orthonormal basis. The orthogonal chaotic vectors solves the estimation problem due to cross correlation and the use of constant energy chaotic sequences i.e., normalized chaotic sequences solve the estimation problem caused due to auto correlation.

1.4 Previous Work

A novel multi-level chaos based communication scheme using orthogonal chaotic basis was first proposed in [20], called quadrature chaos shift keying (QCSK). In QCSK scheme orthogonal chaotic basis are generated using Fourier expansion or Hilbert transform. In [21] it is proved that the Fourier expansion method used to generate orthogonal chaotic basis sets cannot be used to generate orthogonal chaotic basis sets greater than two. Thus, QCSK scheme has a dimensionality problem i.e., it is not applicable only for the dimension greater than two. Therefore, to solve this dimensionality problem in [21], a novel multi-level chaotic communication system was proposed called Orthogonal chaotic vector shift keying (OCVSK). In [21] orthogonal chaotic basis sets are generated using Gram-Schmidt orthonormalization process m-orthonormal chaotic basis sets are generated and used to encode m different symbols for increased data transmission rates.

1.5 Contributions

This thesis mainly concentrates on the simulation and analysis of bit error rate performance of the coherent and non-coherent multi user chaotic communication systems when orthogonal chaotic vectors are used for spreading.

1.5.2.Coherent multi user chaotic communication system

The performance analysis of multi user chaotic communication using orthogonal chaotic vectors is tested by first considering a perfectly synchronised coherent receiver which is an ideal scenario. The performance of the multi user chaotic communication system with coherent receiver is tested through simulations by considering different transmission channel. Theoretical equations for bit error rate are derived for the case of AWGN and flat fading channels. For the case of frequency selective fading, time varying frequency selective fading and fast frequency fading channel the bit error rate analysis is done through simulation.

1.5.3.Non-coherent multi user chaotic communication systems

The performance of non-coherent multi user chaotic communication system with adaptive multi user receivers is analysed through simulation and analytical expressions by considering AWGN channel.

1.6 Overview of dissertation

Goal of dissertation: The goal of this dissertation is to document the bit error rate performance analysis of the multi user coherent and non-coherent multi user chaotic communication systems using orthogonal chaotic vectors. The simulation results and equations for bit error rates presented in this dissertation provides the lower bound bit error rate limit for coherent and non-coherent multi user chaotic communication systems.

Chapter 2: In second chapter some chaos based communication systems like Antipodal Chaos Shift Keying (ACSK), multi user ACSK, Differential Chaos Shift Keying (DCSK) are dealt in brief. This helps in understanding succeeding chapters.

Chapter 3: In this chapter the concept of Orthogonal Chaotic Vectors (OCV) is introduced and its application to Multi user chaotic communication system with coherent receiver is discussed. And also the performance of the multi user chaotic communication system using OCV is analyzed through simulation and analytical equations by considering AWGN channel.

Chapter 4: This chapter deals with the simulation and analysis of multi user chaotic communication systems using orthogonal chaotic vectors with coherent receivers under multi path fading channel conditions with/without rake receivers.

Chapter 5: In this chapter the application of OCV to non-Coherent multi user chaotic communication system with different types of adaptive multi user receivers (LMS/NLMS/RLS/CSE based receivers) are discussed. The BER performance of the system is evaluated using simulation as well as analytical equations.

Chapter 6: A conclusion is drawn in this last chapter, followed by discussions pointing about future works related to the results presented in the dissertation.

2. Chaotic Digital Communication Systems

Ever since, Pecorra & Carroll have shown that the coupled chaotic systems can be synchronized [22,23], the research interest in the area of chaos application in communication has increased. In past two decades, an extensive research has been done in the area of chaotic communication and many analog [24,25,26] and digital [12] [27,16] [17,18] [28,29,30]chaotic communications systems are proposed and analysed.

Digital chaotic communication systems can be divided in to two types, coherent and non-coherent chaotic communication system. In coherent communication system the information at the receiver is recovered by correlating the received signal with that of locally generated chaotic sequence through synchronization process e.g. Antipodal Chaos Shift Keying (ACSK), Multi User ACSK [31] etc. Coherent chaotic communication is not a feasible solution for practical implementation due to the difficulty in synchronization process. Non-coherent chaotic communication systems are practically realizable. In non-coherent communication systems a replica of the chaotic sequence used to modulate the information bit is also transmitted as reference. Some of the non-coherent chaotic communication systems are, differential chaos shift keying (DCSK) [31], Generalized chaos delay shift keying (GCDSK) [32], Multiple access-DCSK [33] etc.

2.1 Chaotic Signals versus PN Sequences

M-sequences and Gold sequences are the most popular conventional spreading sequences in spread spectrum systems. M-sequences, also known as maximum-length sequences are sequences that repeat every 2n-1, where n is an integer. These sequences can be implemented using shift registers. Similarly, Gold codes are constructed from a preferred pair of m-sequences, by the element-by-element multiplication of one m-sequence with every time shift of the second m-sequence.

2.1.1 Auto-Correlation

One of the most important characteristics of PN sequences is its auto-correlation properties. At the receiver, the received signal is mixed with a locally generated PN sequence. This must result in maximum auto-correlation, or signal strength at the point of synchronization for the best decision to be made on the incoming signal.

Fig. 2.1 and Fig. 2.2 show, the autocorrelation of a 31-bit m-sequence and Gold codes respectively. Fig. 2.3 shows the autocorrelation of length L = 31 chaotic sequence. M- sequences are said to have very good autocorrelation properties because of their balance property, which means that the sequence will have exactly one more high bit than low bit. This is the closest a sequence consisting of an odd number of 1 s can get to a zero average. The autocorrelation is given as N when the time lag is 0 and given as -1/N at all other times. Gold sequences can be unbalanced and thus causing some spikes in their autocorrelation properties.

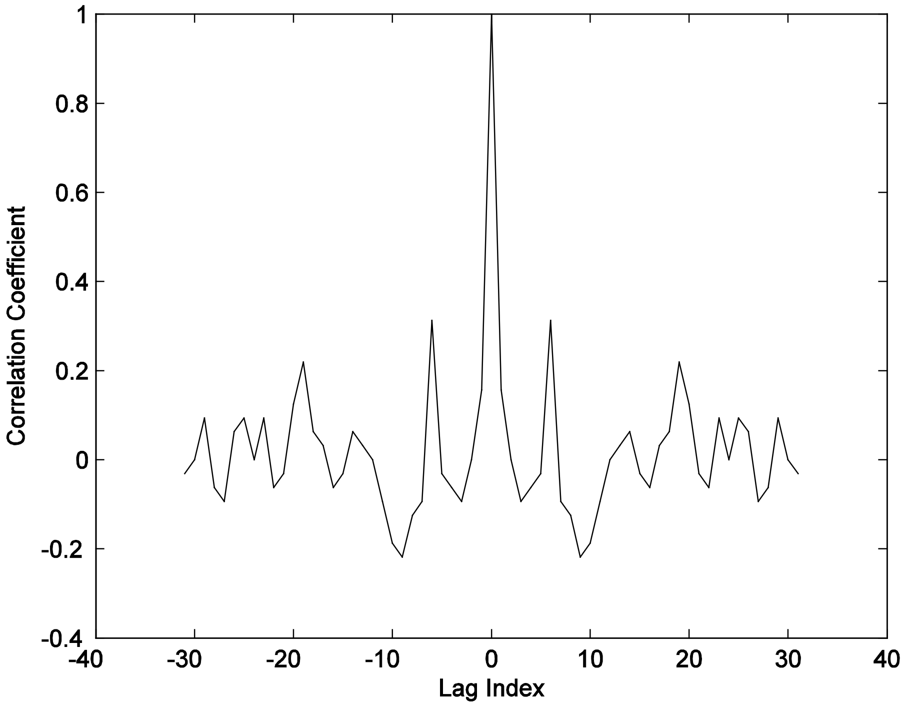


Fig. 2.1Auto Correlation of m sequence

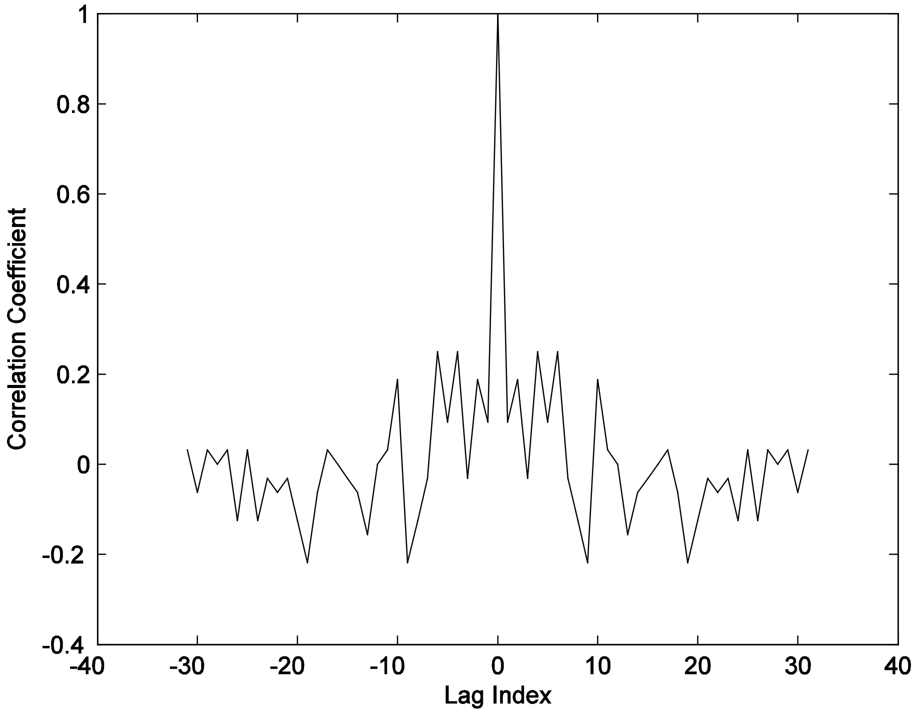


Fig. 2.2Auto Correlation of Gold sequence

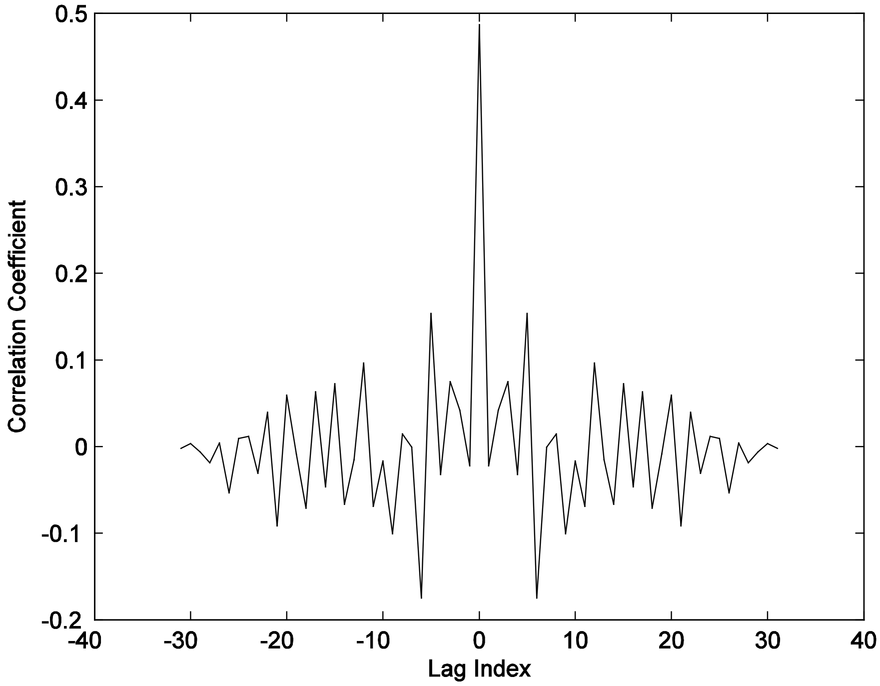


Fig. 2.3 Auto Correlation of Chaos sequence

2.1.2 Cross-Correlation

Another desired property in PN sequences is minimum cross-correlation. In a spread- spectrum system with multiple users, when the received signal is mixed with the locally generated PN sequence, it must result in minimum signal strength.

This would ensure the receiver would be able to differentiate between the transmitted PN sequence and PN sequences of other users.

Fig. 2.4, Fig. 2.5 and Fig. 2.6 show the cross correlation of m-sequences, Gold and Chaos sequences. Studies show that m-sequences have very bad cross correlation properties. For much longer sequences, Gold codes have much lower cross-correlation than m-sequences, and chaotic sequences have been known to achieve much lower cross-correlation bounds than Gold codes.

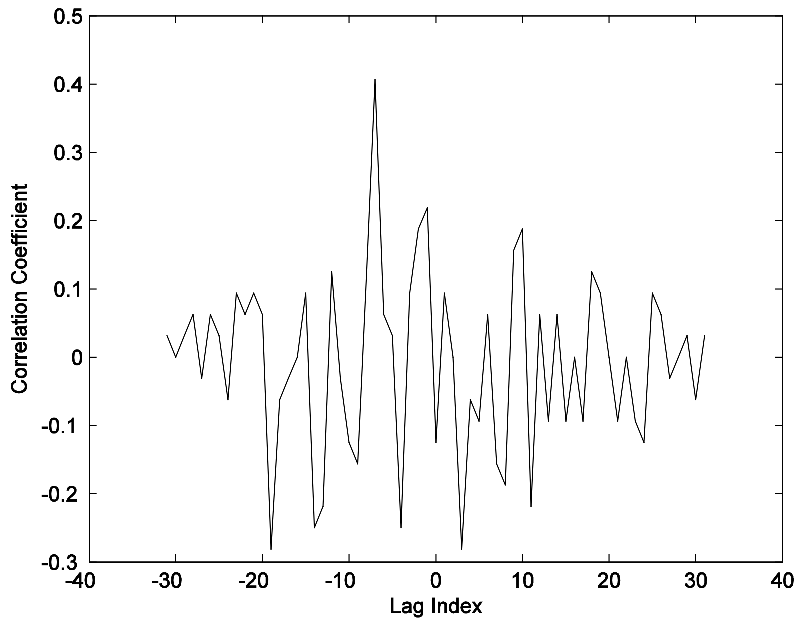


Fig. 2.4Cross Correlation of m sequences

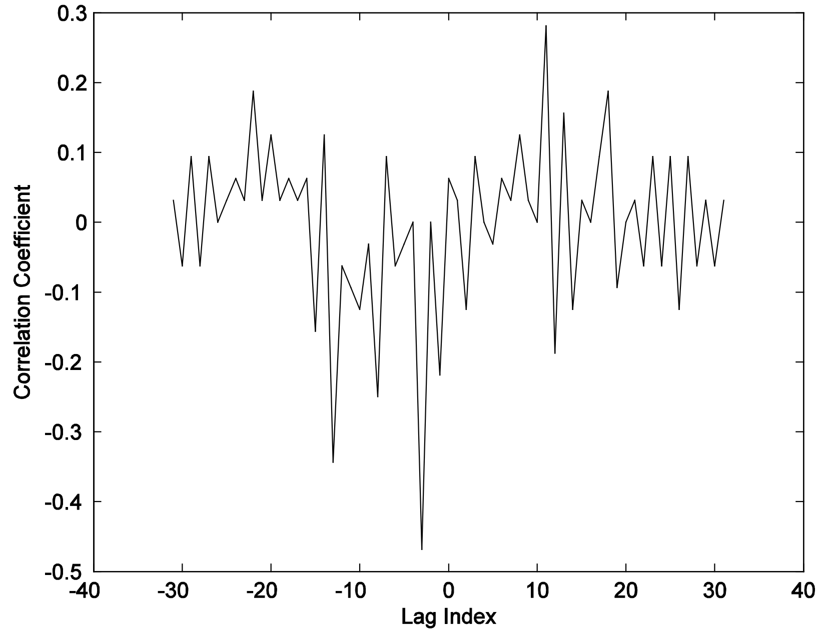


Fig. 2.5Cross Correlation of Gold sequences

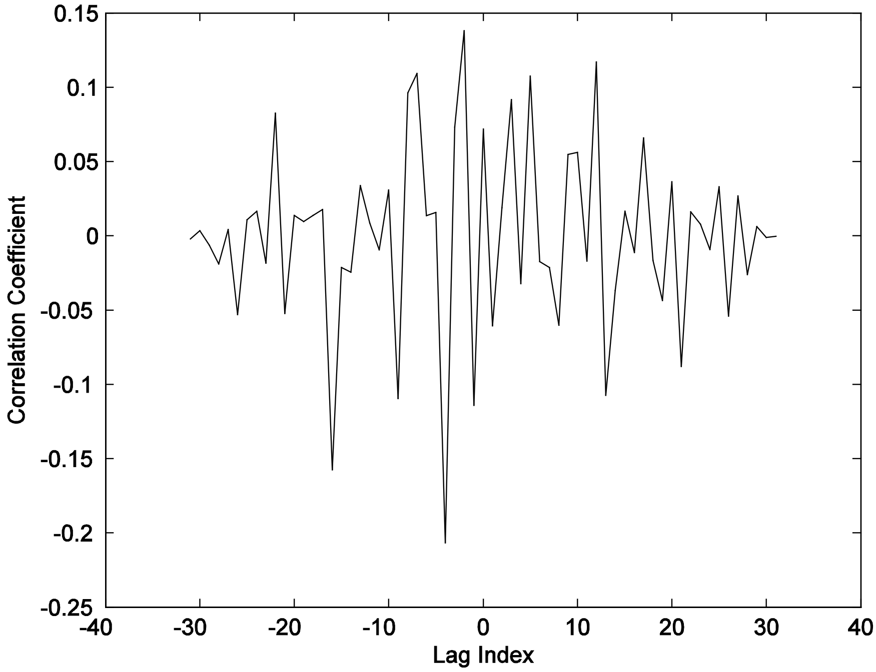


Fig. 2.6Cross Correlation of Chaos sequences

2.1.3 BER Performance

In digital communication, the bit-error rate (BER) is the number of errors in decoding a received bit, divided by the number of transmitted bits during a certain time interval. The errors can be caused due to noise in the channel or interference from other users on the same channel. The bit error rate is one of the best performance measures in comparing different communication systems. Following is a comparison of the BERs of conventional spreading sequences to chaotic spreading sequences. Figure 2.7 and 2.8 show the multiuser performance of length 128 of these sequences in AWGN Channel. Gold sequences seem to perform the best, followed by chaotic sequences and finally m-sequences. If performance is not top priority but rather security, chaotic sequences seem to be the better choice.

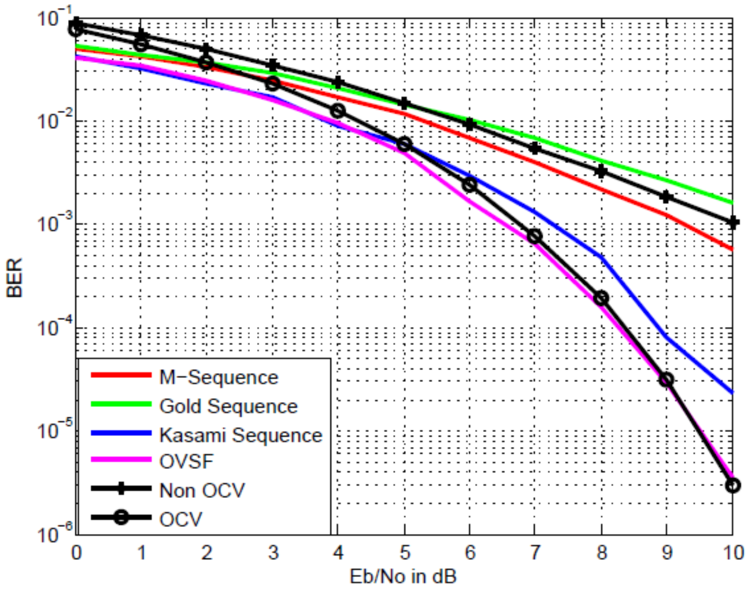


Fig. 2.7 16-User: Chaos vs. Conventional Spreading Sequences

2.1.4 Multiuser Capability

Table 2.1 Comparison of Multi user Capability of Different Spreading Sequences **[34]**

N	$m - sequence$	$Gold$	$Chaos$
7	2	9	>>9
15	2	17	>>17
31	6	33	>>33
63	6	65	>>65
127	18	129	>>129

We now come to the main advantages of Chaos sequences. The first of these is its higher multiuser capability. As shown in the table above, m-sequences have certain properties and only a limited number can be generated given a certain length of the sequence. Gold Codes have a much greater benefit over m-sequences in the number of possible sequences that can be generated. Gold codes are generated by carefully selecting certain m-sequences that exhibit a criterion of correlation properties. The maximum number of Gold codes that can be generated for a given length of sequence N is $N + 2$ sequences. This is a definite advantage over m-sequences in a multiuser scenario, when many users are required on the system. On the other hand, Chaos sequences have a far greater amount of possible sequences that can be generated. Because of its unique property of sensitivity to initial conditions and a large number of chaotic waveform equations, the number of chaotic sequence though not infinite, but a much larger amount compared to m-sequences and even Gold codes. A drawback of this is that this might increase the interference between the different users on the system using chaotic sequences.

2.1.5 Security

We finally come to the main advantage of chaotic sequences over conventional PN sequences, its high security capability and low probability of interception. When it comes to security of a system in comparing these 3 spreading sequences, chaotic sequences definitely are much more secure than conventional PN sequences. This is mostly because of their aperiodicity and their inherent feature of sensitivity to initial conditions. Figure 2.7 shows the waveform of a first order chaotic dynamical system known as a Logistic Map. We can notice from the figure the noise like characteristics of the chaotic signal. Also, we notice that the signal is bounded within the range [-1, +1]. The figure specifically shows the unique feature of chaotic signals' sensitive dependence on initial conditions. Generating the Logistic Map from two slightly different initial conditions, i.e. (x_0 = 0.1, 0.100001) we can notice that after a few iterations the system diverges exponentially.

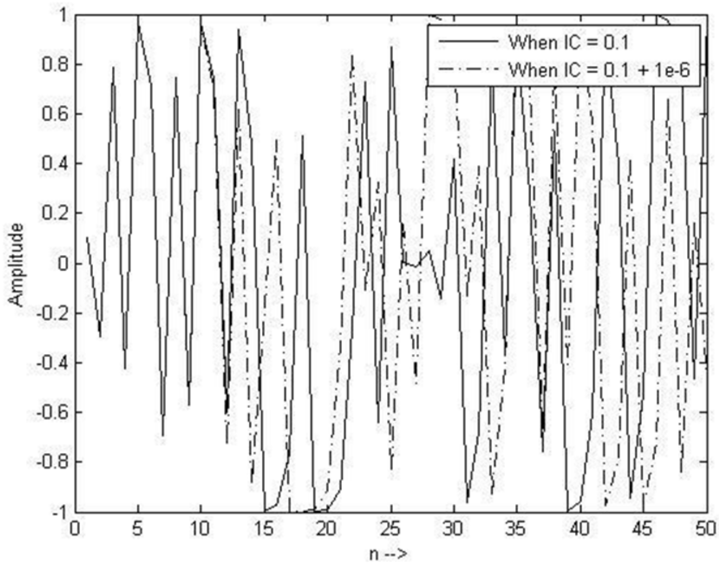


Fig. 2.8 Sensitivity to Initial Conditions for Chaos Sequences

The primary reason for choosing chaotic waveform as a means of transmitting data is the high security of chaos sequences and the low probability of interception of these sequences. The fact is that, unless the exact initial condition and the same chaotic waveform equation are known to the intruder, it is impossible to recover the signal.

2.2 Chaotic digital communication systems with coherent receiver

2.2.1 Single User Antipodal Chaos Shift Keying

Single user antipodal chaos shift keying is one of the simplest form of coherent chaotic communication system. Fig. 2.8 and Fig. 2.9 depicts the coherent antipodal CSK modulator and demodulator [18]. In a coherent antipodal CSK modulator, the chaotic signal *c(t)* is first generated. If a "+1" is transmitted, the chaotic signal will be sent. If a "−1" is transmitted, an inverted copy of the chaotic signal is used as the transmitted signal. Hence, the transmitted signal can be expressed as

$$s(t)=\begin{cases} c(t) & \text{when symbol "+1" is transmitted} \\ -c(t) & \text{when symbol "−1" is transmitted} \end{cases}$$

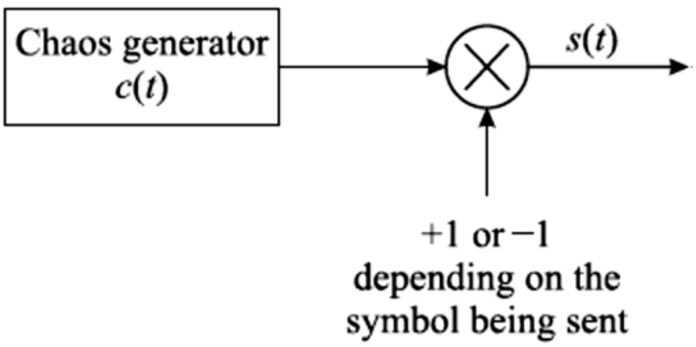


Fig. 2.9. Transmitter structure of Antipodal chaos shift keying

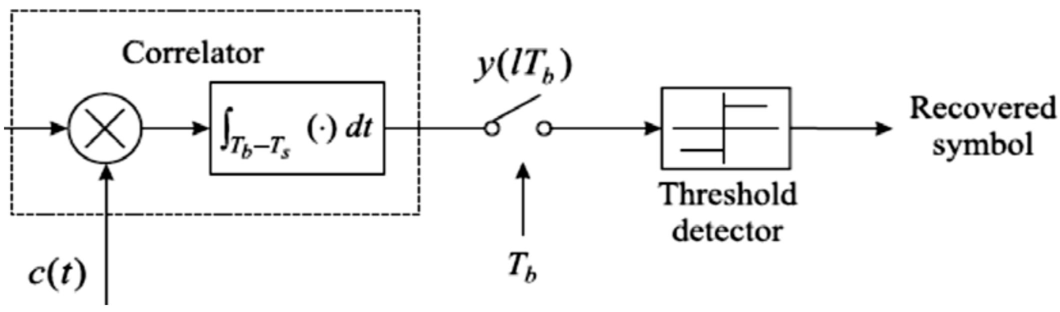


Fig. 2.10. Receiver structure of the antipodal chaos shift keying

Assuming that the transmitted signal is corrupted by additive noise, the received signal is given by

$$r(t) = s(t) + \xi(t)$$

where ξ(t) denotes the noise signal. At the receiver, it is assumed that the chaotic signal generated is perfectly synchronized with the transmitter. Then, the output of the correlator is given by

$$y(t) = \int r(t)c(t)\,dt$$

The output of the correlator is compared with the threshold (zero in this case) to determine whether a "+1" or "−1" has been received. If the correlator output is larger than zero, a "+1" is detected. Otherwise, a "−1" is decoded.

Assuming perfect synchronization and constant bit energy the bit error rate for the coherent antipodal chaos shift keying (ACSK) over AWGN is given by[3],

$$BER = 0.5erfc\left(\sqrt{\frac{E_b}{N_o}}\right)$$

This is equal to the bit error rate of the single user binary phase shift keying (BPSK) [35].

2.2.2 Multi user Antipodal Chaos Shift Keying

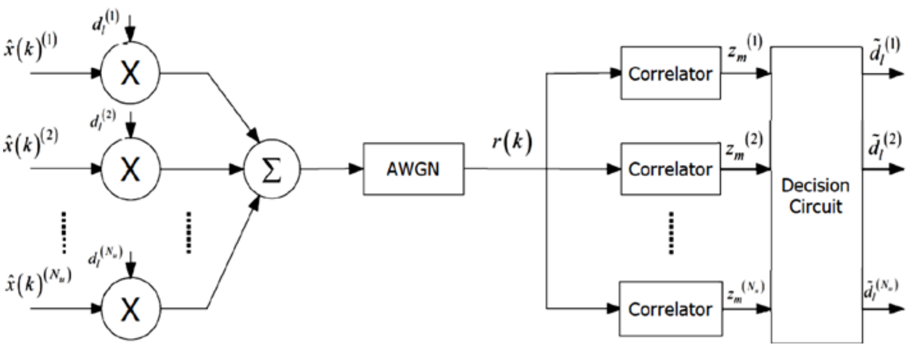


Fig. 2.11 Transmitter and receiver structure of multi user antipodal chaotic communication system

The coherent antipodal chaos-shift-keying communication system with *Nu* users [28], as shown in Fig. 2.10. A discrete-time approach will be adopted in the following analysis. Suppose the *i*th user is transmitting a binary symbol $d^{(i)}_l$ during the l^{th} bit duration, and the symbol $d^{(i)}_l$ is either "+1" or "−1", each with equal probability. Also, the symbols sent by different users are independent of one another. Essentially, there are *Nu* chaos generators corresponding to the *Nu* different users, and the i^{th} chaos generator produces the chaotic samples $\{x^{(i)}{}_k\}$, which is used to spread the binary symbol sequence $\{d^{(i)}{}_l\}$. We assume that the chaotic sequences are generated independently of one another. Define the spreading factor, β, as the number of chaotic samples used to transmit one binary symbol. We also assume that the mean value of each of the chaotic sequences is zero in order to avoid transmitting any dc component which is a waste of power.

Assuming that the chaotic sequences generated at the receiver are perfectly synchronized and constant bit energy. The bit error rate for the coherent multi user antipodal chaos shift keying system is given by [31],

$$BER = \frac{1}{2} erfc\left[\left(\frac{2(N_u - 1)}{\beta} + \left(\frac{E_b}{N_o}\right)^{-1}\right)^{-\frac{1}{2}}\right]$$

2.3 Chaotic digital communication systems with non-coherent receiver

2.3.1 Single user Differential Chaos Shift Keying

When the CSK signals are decoded based on the estimation of the bit energy, the threshold of the detector should shift with the noise level. Otherwise, lots of errors would occur. To overcome this issue, differential chaos-shift-keying (DCSK) modulation scheme has been proposed [63].

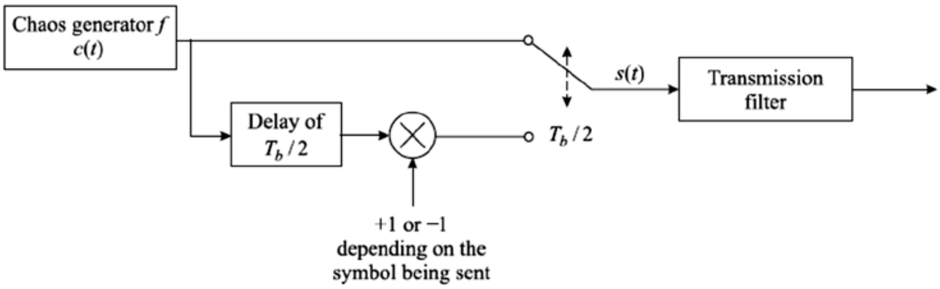


Fig. 2.12. Transmitter structure of single user differential chaos shift keying

Fig. 2.11 shows the block diagram of a DCSK system. In DCSK modulation, each transmitted symbol duration is divided into two identical time slots. The first one is used to transmit a chaotic reference signal while the second time slot sends an information-bearing signal. During the second time slot, if a symbol "+1" is to be transmitted, the chaotic reference signal is repeated; if the symbol "−1" is to be transmitted, an inverted copy of the reference signal will be sent. Hence, the transmitted signal is given by

$$s(t)=\begin{cases} c(t) & for\,(l-1)T_b \le t < (l-1/2)T_b \\ c(t-T_b/2) & for\,(l-1/2)T_b \le t < lT_b \end{cases}$$

If the l^{th} symbol is "+1".

$$s(t)=\begin{cases} c(t) & for\,(l-1)T_b \le t < (l-1/2)T_b \\ -c(t-T_b/2) & for\,(l-1/2)T_b \le t < lT_b \end{cases}$$

If the l^{th} symbol is "−1", where *c(t)* denotes the chaotic reference signal.

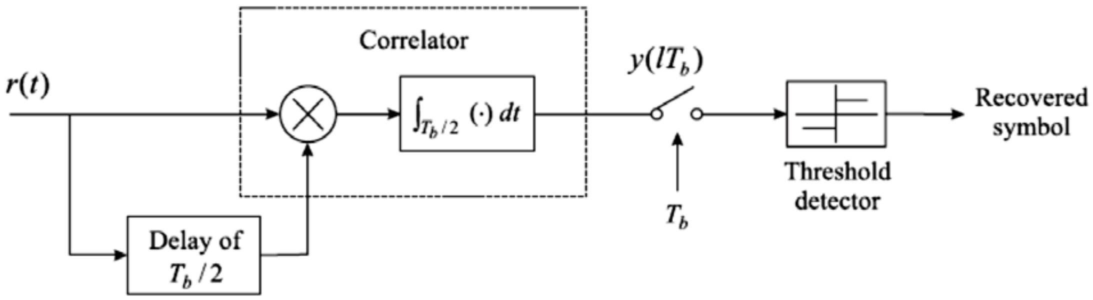


Fig. 2.13. Receiver structure of single user differential chaos shift keying

At the receiver (Fig. 2.12), the received signal in the second half symbol duration will correlate with that in the first half symbol duration.

Assuming that the energy per bit of the transmitted bit is constant the bit error rate of the differential chaos shift keying system is given by [18],

$$BER = 0.5erfc\left(\sqrt{\frac{E_b}{2N_o}}\right)$$

The extensive bit error rate analysis of differential chaos shift keying (DCSK) over AWGN [36] and multipath fading channel can be found in [36] [37] [38]. Some modified structures for DCSK scheme [39] [40], has also been proposed to improve the performance.

2.3.2 Multiple access techniques for non-coherent chaotic communication systems

A novel multiple access techniques for non-coherent communication system were first presented in [33]. A training assisted non coherent multi user chaotic communication system was first presented in [17], where Walsh codes are used to modulate the reference chaotic sequences followed by modulating the chaotic sequences by information bit. Later the improved multiple access technique for non-coherent chaotic communication using adaptive receivers was proposed by W.M Tam, *et. al.,* [41] and it was called as “A multiple access scheme for chaos based digital communication system utilizing transmitted reference”. In this scheme, the data is transmitted in the form of frame. The frame is divided in to two parts. The first part containing the chaotic sequences modulated using training data and the second part of the frame contained the chaotic sequences modulated using information bits. With the help of training data and the adaptive algorithms the chaotic sequence is estimated and then it is used to extract the information contained in the second part of the frame. In [41] two types of adaptive multi user receivers are used namely, 1. LMS/NLMS based multi user receiver and 2. Inverse Averaging (IA) receiver. In LMS/NLMS based receivers with the help of training data and LMS/NLMS algorithm the chaotic sequence is estimated and the estimated chaotic sequence is used to extract the information bit. Whereas in IA based receiver the training portion of the received frame is multiplied by the training sequence and chaotic sequence from each slot is added and finally averaged to get the estimated chaotic sequence.

The multiple access technique proposed in [41] was limited to the synchronous transmission so, Coulon, *et. al.*[[42],extended the work to generalize the multiple access scheme for asynchronous transmission also. Coulon, *et.al.,* [42] generalized the work of [41] for asynchronous transmission and used four types of adaptive multi user receiver namely, 1. LMMSE Receiver 2. LMS/NLMS based multi user receiver and 3. CSE based multi user receiver and 4. CSE-MMSE based multi user receiver. The function of CSE based multi user receiver is similar to IA based receiver presented in [23-24]. In CSE-MMSE based multi user receiver the chaotic sequence is first estimated using CSE algorithm and MMSE is applied on estimated chaotic sequences in order to reduce multiple access interference (MAI).

3. Multi User Chaotic Communication System Using Orthogonal Chaotic Vector with Coherent Receiver

Due to non-zero cross-correlations between the chaotic spreading signals, the interference caused by undesirable users increases as the number of simultaneous users increases. Since the correlator-type receiver is not designed to combat such interference, multi user interference degrades the bit error performance of all users. Such multi user interference (MUI) limits the performance of the system even under a noiseless environment. In order to mitigate the effect of MUI many multiuser detectors have been used for improving the bit error performance [43]. Another way to eliminate multi user interference is to use the spreading sequences which are perfectly orthogonal to each other [44], e.g. Walsh-Hadamard codes in which case there will be no interference between users in synchronous transmission environment. The orthogonality of the spreading sequences is lost under asynchronous transmission scenario and also in multi path environment [45].

A novel multi-level chaos based communication scheme using orthogonal chaotic basis is proposed in [20], called quadrature chaos shift keying (QCSK). In QCSK scheme orthogonal chaotic basis are generated using Fourier expansion or Hilbert transform. In [21], it is shown that the Fourier expansion method used in QCSK scheme to generate orthogonal basis cannot be extended for dimensions greater than two i.e., generation of number orthogonal chaotic basis sets greater than two. Motivated by the concept presented in [20], an orthogonal chaotic vector shift keying (OCVSK) scheme for multi-level communication system with increased data transmission rate and security is proposed in [21]. In the communication scheme presented in [21] orthonormal chaotic basis are generated using gram-Schmidt orthonormalisation process which is not limited to two orthogonal basis as in the case of QCSK.

In this dissertation the application of orthogonal chaotic vectors in multi user chaotic communication is tested and analysed. Since, the spreading sequences are orthogonal they will have zero cross correlation and thus resulting in zero MUI. At the receiver simple correlator based receiver can be used.

In this chapter, the generation of Orthogonal Chaotic Vector (OCV) and its application in multi user chaotic communication system with coherent receiver is discussed. The performance of the system is analysed through simulation and analytical expressions for transmission through AWGN channel.

3.1 Orthogonal Chaotic Vectors (OCV)

Orthogonal chaotic vectors can be generated using Gram-Schmidt ortho-normalization process. The mean value of chaotic carrier is made equal to zero in order to avoid unwanted dc power transmission. Gram-Schmidt ortho-normalization process [35] for N_u number of sequences is given by

$$\hat{x}(k)^{(p)} = \frac{x(k)^{(p)} - \sum_{q=1}^{p-1}\left[\sum_{k=1}^{\beta} x(k)^{(p)}\,\hat{x}(k)^{(q)}\right]\hat{x}(k)^{(q)}}{\left(\sum_{k=1}^{\beta}\left[x(k)^{(p)} - \sum_{q=1}^{p-1}\left[\sum_{k=1}^{\beta} x(k)^{(p)}\,\hat{x}(k)^{(q)}\right]\hat{x}(k)^{(q)}\right]^2\right)^{1/2}} \tag{3.1}$$

Where p = 2, 3. . . Nu, For p = 1

$$\hat{x}(k)^{(1)} = \frac{x(k)^{(1)}}{\left(\sum_{k=1}^{\beta}\left[x(k)^{(1)}\right]^2\right)^{1/2}} \tag{3.2}$$

Where $x(k)^{(i)}$ is the chaotic vector for i^{th} user and N_u is the number of users. Through Gram-Schmidt ortho-normalization, Nu number of vectors of length β is represented ortho-normally in $\mathbf{R}^{\beta}$ Euclidean space. So, for all N_u number of sequences to be ortho-normal after gram-schmidt process the spread factor, β greater than number of users, N_u . Therefore, for the values of spread factor,β selected less than the number of users, N_u the chaotic vectors will not be ortho-normal to each other, thus causes MUI.

The auto correlation and cross correlation profiles of orthogonal chaotic vectors for four user case is shown in Fig. 3.1 and power spectral density of OCV in Fig. 3.2. From Fig. 3.1 and Fig. 3.2 it is observed that the application ortho-normalization process on chaotic vectors will retain wide band frequency spectrum characteristic with improved correlation property (because of orthogonality).

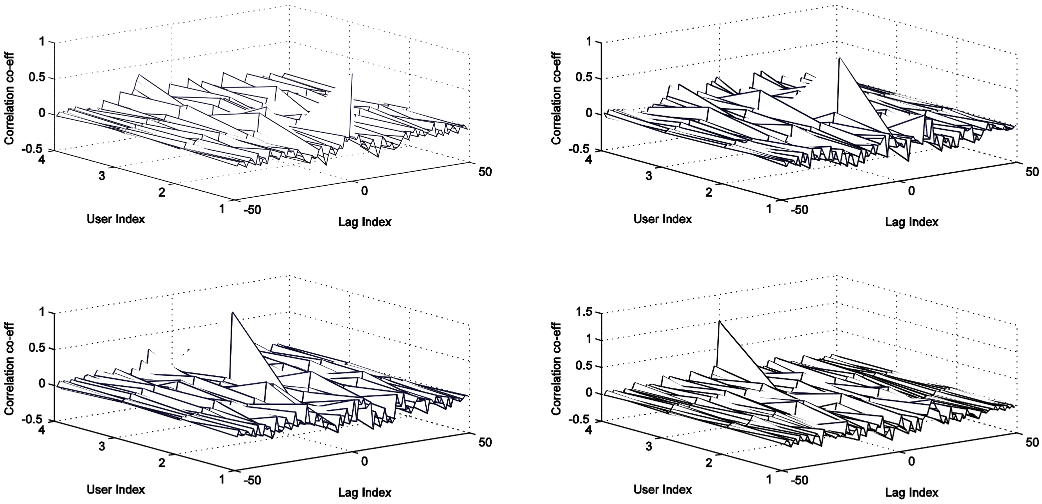


Fig. 3.1. Auto Correlation and Cross Correlation profile of Orthogonal Chaotic Vector in 4 users system ((a) – (d) : User 1 – User 4)

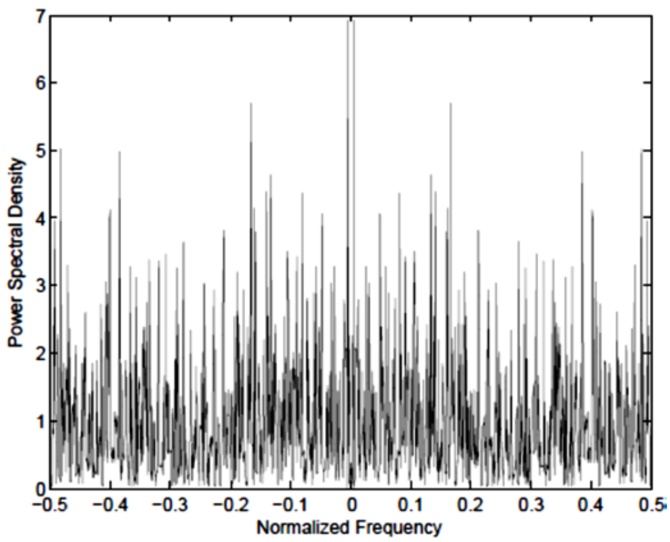


Fig. 3.2. Power Spectral Density of Orthogonal Chaotic Vector

3.2 System Architecture

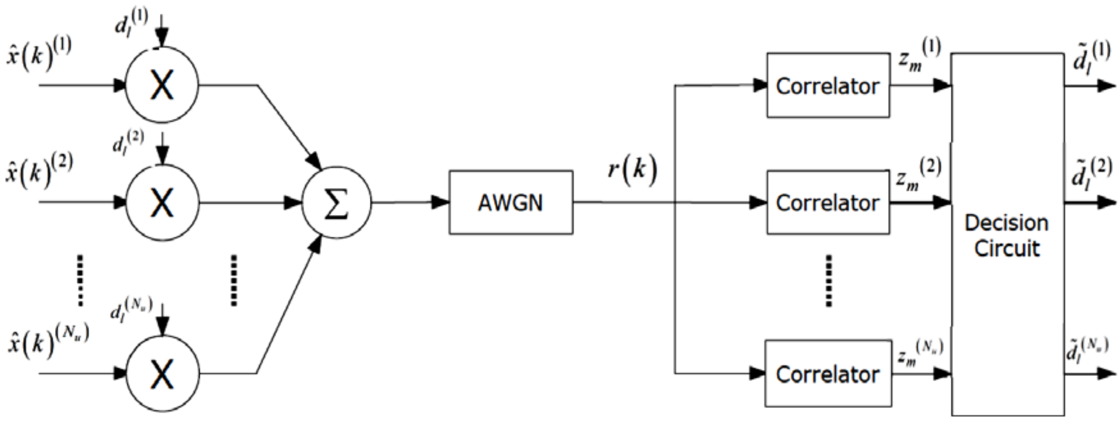


Fig. 3.3 Transmitter and Receiver structure of the Multi user Chaotic Communication system

The transmitter and receiver structure of multi user chaotic communication system [3] is shown in Fig. 3.3. The transmitter contains N_u number of chaotic signal generators, where N_u is the number of users. The chaotic generators used to generate carriers may be from different chaotic maps or from the same map but with different initial conditions. If $\hat{x}(k)_m^{(i)}$ is the orthogonal chaotic vector used to spread the m^{th} data bit of i^{th} user and defining the number of chaotic samples used to transmit single binary bit as the spreading factor, β. It is then modulated by data sequence $d_m \in \{-1, +1\}$. The modulated sequence is given by,

$$v(k)_m^{(i)} = \begin{cases} E_b^{1/2}\hat{x}(k)_m^{(i)} \; if \; d_m^{(i)} = +1 \\ -E_b^{1/2}\hat{x}(k)_m^{(i)} \; if \; d_m^{(i)} = -1 \end{cases} \tag{3.3}$$

Where, E_b is the transmitted bit energy.

The transmitted signal $s(k)_m$ for m^{th} transmitted data bit is the sum of modulated ortho normalized chaotic signals $\hat{x}(k)_m^{(i)}$ of each user given by,

$$s(k)_m = \sum_{i=1}^{N_u} v(k)_m^{(i)} \tag{3.4}$$

A correlator based receiver is used assuming perfect synchronization.

3.3 Bit Error Rate Performance Analysis over AWGN Channel

In literature, many methods have been discussed for the computation of accurate and approximate bit error rates for chaotic communication [31] [46] [47] [48]. Among these the Gaussian approximation method [31] [37] is the simplest method. In this dissertation equations for bit error rate are derived using Gaussian approximation method considering its simplicity.

Assuming that the signal is corrupted only due to AWGN, received signal for m^{th} transmitted bit the received signal $r_m(k)$ can be represented as

$$r(k)_m = \sum_{i=1}^{N_u} v(k)_m^{(i)} + \xi(k) \tag{3.5}$$

Where $\xi(k)$ represents the additive white Gaussian noise with zero mean and variance $N_o/2$. At receiver it is assumed that exact replica of chaotic vector is available for despreading and it is exactly synchronized with the transmitter then, the

m^{th} decoded symbol for the j^{th} user, denoted by $\tilde{d}_m^{(j)}$, is determined according to the rule:

$$\tilde{d}_m^{(j)} = \begin{cases} +1 \ \ if \ \ z_m^{(j)} = \sum_{k=1}^{\beta} r(k)_m \, \hat{x}(k)_m^{(j)} \ > \ 0 \\ -1 \ \ if \ \ z_m^{(j)} = \sum_{k=1}^{\beta} r(k)_m \, \hat{x}(k)_m^{(j)} \ \leq \ 0 \end{cases} \tag{3.6}$$

Correlator output of j^{th} is given by, $z_m^{(j)} = D + MUI + \xi$

$$D = d_m^{(j)} \sum_{k=1}^{\beta} \left[E_b^{\ 1/2} \hat{x}(k)_m^{(j)} \right]^2$$

$$MUI = \sum_{i=1, i \neq j}^{N_u} d_m^{(i)} \sum_{k=1}^{\beta} E_b \left(\hat{x}(k)_m^{(i)} \, \hat{x}(k)_m^{(j)} \right)$$

$$\xi = \sum_{k=1}^{\beta} E_b^{\ 1/2} \xi(k) \hat{x}(k)_m^{(j)}$$

Where, term D is the desired term which has the information bit, MUI is the multi user interference caused by the other users and ξ is the noise term. To simplify the representation the subscript m can be neglected in further equations.

Assuming, that $z^{(j)} \Big| \left(d^{(j)} = -1 \right)$ and $z^{(j)} \Big| \left(d^{(j)} = +1 \right)$ are random variables. According to central limit theorem for sum of large set of these random variables leads to Gaussian distribution. Therefore, BER for j^{th} user can be written as

$$BER^{(j)} = \frac{1}{2} erfc \left(\frac{E\left(z^{(j)} | d^{(j)} = +1 \right)}{\left(2 \operatorname{var}\left(z^{(j)} | d^{(j)} = +1 \right) \right)^{1/2}} \right) = \frac{1}{2} erfc \left(\frac{-E\left(z^{(j)} | d^{(j)} = -1 \right)}{\left(2 \operatorname{var}\left(z^{(j)} | d^{(j)} = -1 \right) \right)^{1/2}} \right) \tag{3.7}$$

Where, mean value of $\left(z^{(j)} | d^{(j)} = +1 \right)$ is given by,

$$E\left(z^{(j)} | d^{(j)} = +1 \right) = \beta E_b E\left[\left(\hat{x}(k)^{(j)} \right)^2 \right] \tag{3.8}$$

And variance is given by,

$$\operatorname{var}\left(z^{(j)} | d^{(j)} = +1 \right) = \ E_b^2 \ \operatorname{var}\left[\sum_{k=1}^{\beta} \left[\hat{x}(k)^{(j)} \right]^2 \right] + \ E_b^2 \ \operatorname{var}\left[\sum_{i=1, i \neq j}^{N_u} \sum_{k=1}^{\beta} \left(\hat{x}(k)^{(i)} \, \hat{x}(k)^{(j)} \right) \right]$$
$$+ \beta E_b E\left[\xi(k)^2 \right] E\left[\left(\hat{x}(k)^{(j)} \right)^2 \right] \tag{3.9}$$

Where,

$$E\left[\hat{x}(k)^{(j)} \right] = 0 \tag{3.10}$$

$$E\left[\left(\hat{x}(k)^{(j)}\right)^2\right] = \frac{\psi}{\beta} = P_s \tag{3.11}$$

$$\operatorname{var}\left[\hat{x}(k)^{(j)}\right] = \frac{\psi}{\beta} \tag{3.12}$$

$$\operatorname{var}\left[\sum_{k=1}^{\beta}\left[\hat{x}(k)^{(j)}\right]^2\right] = 0 \tag{3.13}$$

$$\operatorname{var}\left[\sum_{i=1, i\neq j}^{N_u}\sum_{k=1}^{\beta}\left(\hat{x}(k)^{(i)}\,\hat{x}(k)^{(j)}\right)\right] = \begin{cases} 0 & for\ \ \beta > N_u \\ \beta(N_u - 1)\mathrm{E}\left[\left(\hat{x}(k)^{(i)}\right)^2\right]\mathrm{E}\left[\left(\hat{x}(k)^{(j)}\right)^2\right] & for\ \ \beta \le N_u \end{cases} \tag{3.14}$$

$$\operatorname{var}\left[\xi(k)\right] = E\left[\xi(k)^2\right] = \frac{N_o}{2} \tag{3.15}$$

Where, P_s is average power transmitted by each user per bit, E_b is energy per bit and ψ is given by, $\psi = \beta\operatorname{var}[\hat{x}(k)^{(j)}]$. Since, the chaotic vectors are ortho normalized, the value of ψ will be equal to 1.

Using the equations from (3.10) to (3.15) in (3.7), we get

$$BER^{(j)} = \begin{cases} \frac{1}{2}erfc\left[\left(\frac{2(N_u - 1)}{\beta} + \left(\frac{E_b}{N_o}\right)^{-1}\right)^{-\frac{1}{2}}\right] & for\ \ \beta \le N_u \\ \frac{1}{2}erfc\left(\frac{E_b}{N_o}\right)^{\frac{1}{2}} & for\ \ \beta > N_u \end{cases} \tag{3.16}$$

$$BER^{(j)}_{LowerBound} = \frac{1}{2}erfc\left(\frac{E_b}{N_o}\right)^{\frac{1}{2}} \tag{3.17}$$

Equation (3.16) gives the expression for bit error rate of j^{th} user. Since, the bits transmitted by all users are equi-probable the expression for average bit error rate for the multi user chaotic communication system using orthogonal chaotic vector will be,

$$BER = \begin{cases} \frac{1}{2}erfc\left[\left(\frac{2(N_u - 1)}{\beta} + \left(\frac{E_b}{N_o}\right)^{-1}\right)^{-\frac{1}{2}}\right] & for\ \ \beta \le N_u \\ \frac{1}{2}erfc\left(\frac{E_b}{N_o}\right)^{\frac{1}{2}} & for\ \ \beta > N_u \end{cases} \tag{3.18}$$

$$BER_{LowerBound} = \frac{1}{2}erfc\left(\frac{E_b}{N_o}\right)^{\frac{1}{2}} \tag{3.19}$$

From equation (3.18) it is clear that for spread factor, β greater than number of users the bit error rate is independent of number of users, N_u and spread factor, β. Equation (3.19) gives the lower bound for bit error rate for the multi user chaotic communication system with coherent receiver which is equal to the bit error rate for single user BPSK system over AWGN channel.

3.4 Simulation

For generating chaotic vectors chebyshev map of order three (Cubic Map), Renyi Map and Bernoulli shift map are used for all simulation presented in this dissertation, which are given by [3] [20]:

Cubic Map: $$x(k+1) = 4x^3(k) - 3x(k) \tag{3.20}$$

Renyi Map: $$x(k+1) = \mathrm{mod}\left(3x(k)+1, 2\right) - 1 \tag{3.21}$$

Bernoulli Shift Map:

$$x(k+1) = \begin{cases} 1.2x(k)+1 & if \quad x(k) < 0 \\ 1.2x(k)-1 & if \quad x(k) \geq 0 \end{cases} \tag{3.22}$$

Bit error rates are computed by using mont-carlo simulation. To perform monte-carlo simulation 1 million bits are transmitted for each user and at the receiver the number of bits in error are calculated for each user and the bit error rates obtained for each user are averaged to find average bit error rate. The initial condition for chaotic sequence generator is chosen such that the map operates in chaotic regime.

For simulation of multi user chaotic communication system using non-OCV the chaotic vectors are normalized before modulation in order to ensure the constant bit energy of the transmitted signal.

Mixed Analysis Simulation (MAS) [31]

Mixed analysis simulation involves both numerical simulation and analytical method to calculate the bit error rate. In mixed analysis simulation the variance and estimation values are calculated using numerical simulation and the values obtained are substituted in analytical equations to find the bit error rate. To estimate the statistical values i.e., mean and variance of the cross products the method presented in [18] is adapted by using 10^6 samples of chaotic sequences and Gaussian noise sequences.

3.5 Simulation Results

In this section the simulated bit error plots for multi user chaotic communication system using non-orthogonal chaotic vectors, orthogonal chaotic vectors and conventional spreading codes are presented.

In Fig. 3.4 the bit error rates for multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors are plotted for different values of spread factors, β. Bit error rate obtained for orthogonal chaotic vector case is better for spread factor, β equal to 100. Whereas, when spread factor, β is 1000 the BER's for both the cases i.e., non-OCV and OCV have no differences. This is because the chaotic sequences are not perfectly orthogonal but they are quasi orthogonal for small values of spread factor, β. The simulated bit error rates are also compared with the theoretical bit error rate computed from equation (3.18). And it is evident that the simulated bit error rates are in perfect match with bit error rates computed using analytical equations derived. Fig. 3.5 compares the bit error rates of multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors as a function of number of users. Since, the spreading sequences are orthonormal to each other the multi user interferance will be eliminated thus bit error rate will be constant as the number of user is increased for constant bit energy to noise ratio (E_b/N_o) and for spread factor, β being greater than number of users, N_u.

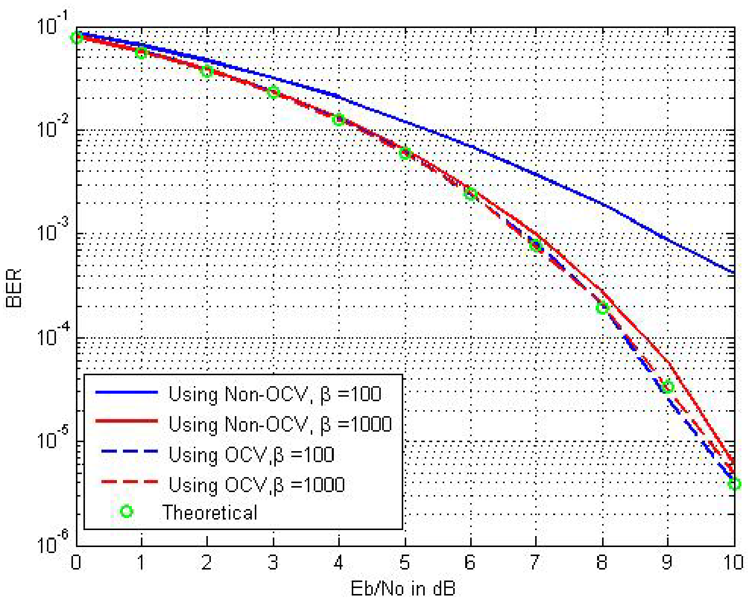


Fig. 3.4 BERs versus E_b/N_o for Number of users = 5

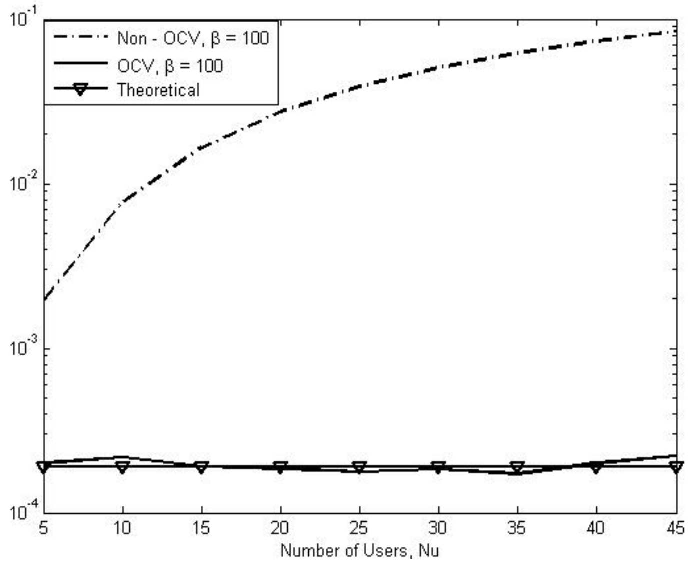


Fig. 3.5. BER's versus Number of users for β = 100 and E_b/N_o = 8

In Fig. 3.6 the bit error rates as a function of bit energy to noise ratio (Eb/No) of multi user chaotic communication system using orthogonal chaotic vectors are compared for different values of spread factor, β and number of users, Nu are plotted to study the accuracy of the bit error rate equation given in equation (3.18) at the discontinuity β = Nu. Here three cases are considered, first when β < Nu then β = Nu and finally β > Nu. For all these three cases the simulated bit error rates are compared with bit error rate computed using equation (3.3.14) and bit error rated obtained from Mixed Analysis Simulation (MAS). In Fig. 3.7 for spread factor, β is 8 and 32 and number of users, Nu is 16 i.e., for the condition β < Nu and β > Nu the simulated bit error rate, theoretical bit error rate and bit error rate computed using MAS are in exact match. But when spread factor, β is 16 and number of users, Nu is 16 i.e., for β = Nu the simulated bit error rate does not match with the bit error rate calculated using equation (3.18) whereas simulated bit error rate and bit error rate computed using MAS are in exact match.

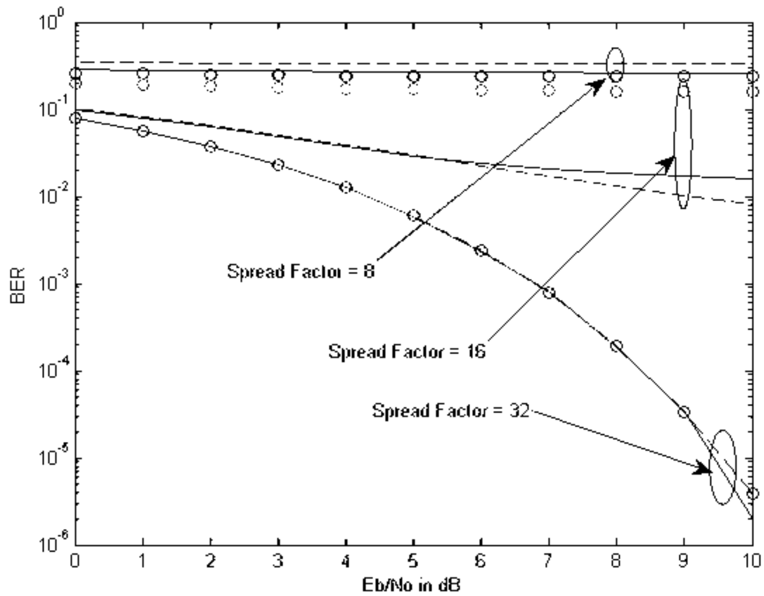


Fig. 3.6. BER versus Eb/No, for Number of users = 16. Solid line: Simulation, Dashed line: MAS, Circles: Theoretical

In Fig. 3.7 the bit error rates as a function of spread factor, β of multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors are compared. For spread factor, β less then number of users, Nu the bit error rate for both cases are same and when spread factor, β is greater than number of users, Nu the bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vector has an excellent performance due to the orthonormal property of the spreading sequence used to spread the information bit. As spread factor, β is increased the bit error rate for non-orthogonal chaotic vector case converges with orthogonal chaotic vector case at about β = 1000 for number of users, N_u = 5. In Fig. 3.8 the simulated bit error rates as a function of spread factor, β of multi user chaotic communication system using orthogonal chaotic vectors are compared with bit error rates computed using equation (3.18) and bit error rates computed using MAS. The simulated bit error rate is in exact match with bit error rate computed using MAS. But the simulated bit error rate does not match with that of theoretical bit error rates computed using equation (3..18) for the values of spread factor, β less than the number of users, N_u.

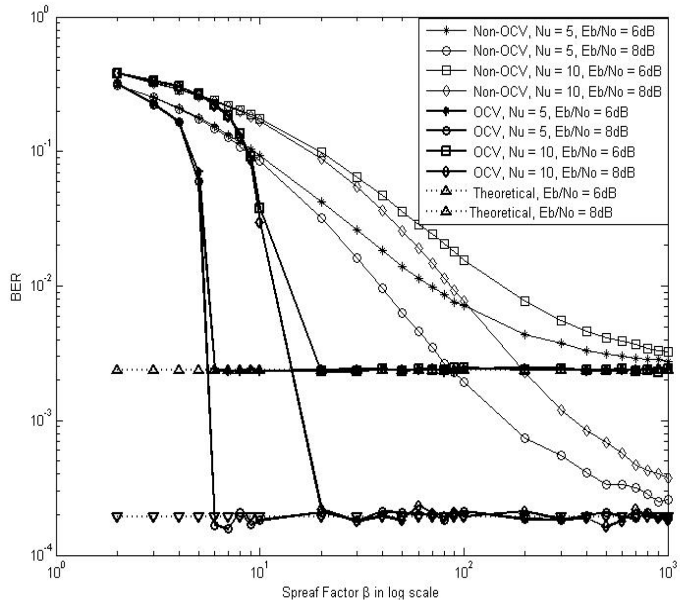


Fig. 3.7 BER v/s Spread Factor β for different Number Of Users, Nu and E_b/N_o Ratio

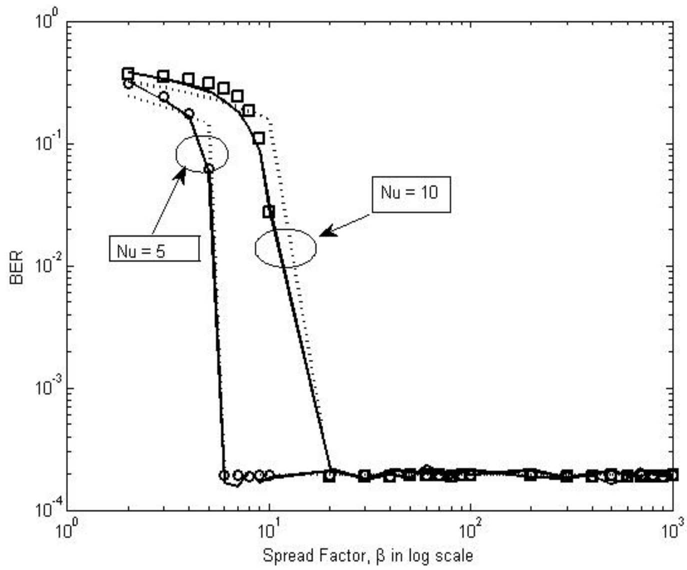


Fig. 3.8 Comparison of BER v/s spread Factor plots for number of users, Nu = 5 and 10. Solid line: Simulated BER, Dotted line: BER computed using MAS, Circle and Square: Theoretical BER

In Fig. 3.9 the simulated bit error rates for multi user chaotic communication system when chaotic sequences generated from different chaotic maps are compared. When ortho-normalized the performance of the multi user chaotic communication system will be independent of the chaotic map used to generate the chaotic sequences whereas in the other case the bit error rate performance of the multi user chaotic communication system depends on the variance of the cross correlation co-efficient which is different for different chaotic map.

In Fig. 3.10(a-d) the comparison of bit error rate plots for multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors are presented. The bit error rates plots of chaotic communication system are also compared with multi user spread spectrum communication system which uses conventional spreading codes such as, m-sequence, gold sequence, kasami sequence and OVSF sequences for spreading the information bit.

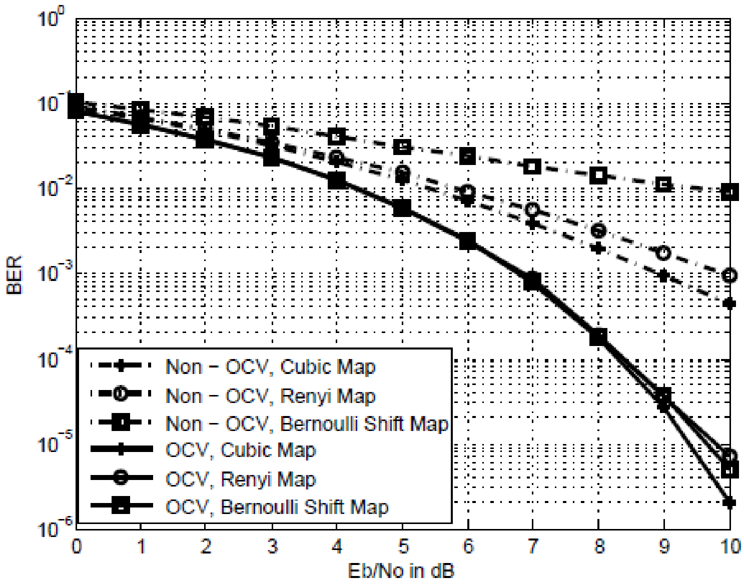
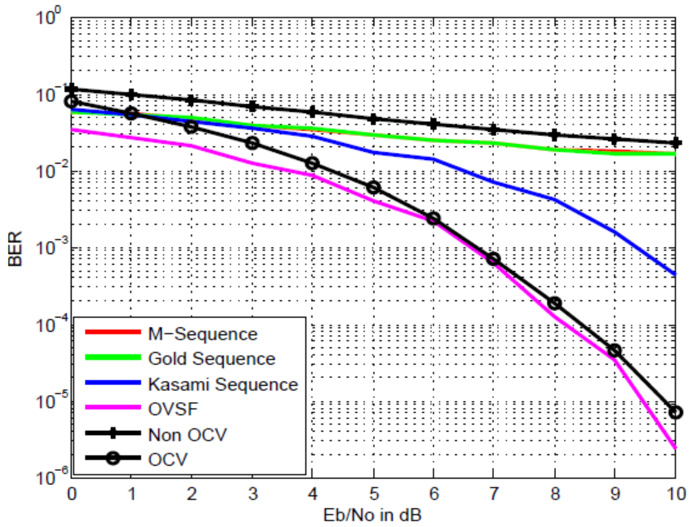
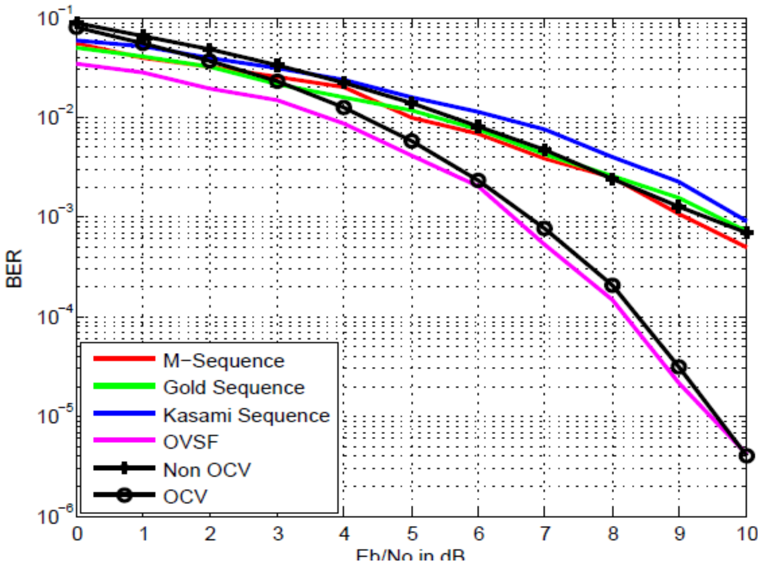


Fig. 3.9 Comparison of BERs v/s Eb/No plots for number of users = 5 and spread factor, β = 100

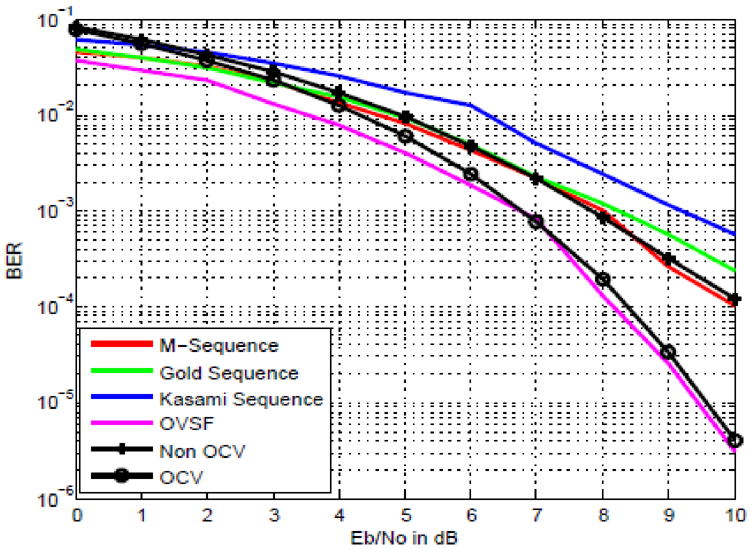


(a) Spread Factor,β = 16 Number of Users, Nu = 4

Fig. 3.10(a-e) Comparison of BER performance of Multi user spread spectrum communication system using chaotic vectors, Orthogonal chaotic vectors and conventional spreading codes, M-Sequence : Polynomial : [11 8 5 2 0],Gold Sequence : Polynomial Pair: [11 8 5 2 0] , [11 10 3 2 0], Kasami Sequence : Polynomial : [11 8 5 2 0] [7]

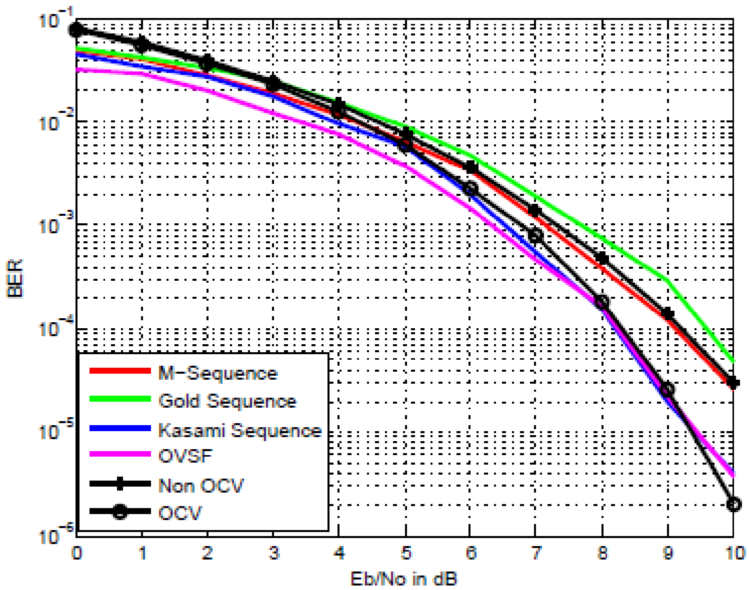


(b) Spread Factor,β = 64, Number of Users, Nu = 4



(c) Spread Factor,β = 128, Number of Users, Nu = 4

Fig. 3.10(a-d) Comparison of BER performance of Multi user spread spectrum communication system using chaotic vectors, Orthogonal chaotic vectors and conventional spreading codes, M-Sequence : Polynomial : [11 8 5 2 0],Gold Sequence : Polynomial Pair: [11 8 5 2 0] , [11 10 3 2 0], Kasami Sequence : Polynomial : [11 8 5 2 0] [7]



(d) Spread Factor,β = 256, Number of Users, Nu = 4

Fig. 3.10(a-e) Comparison of BER performance of Multi user spread spectrum communication system using chaotic vectors, Orthogonal chaotic vectors and conventional spreading codes, M-Sequence : Polynomial : [11 8 5 2 0],Gold Sequence : Polynomial Pair: [11 8 5 2 0] , [11 10 3 2 0], Kasami Sequence : Polynomial : [11 8 5 2 0] [7]

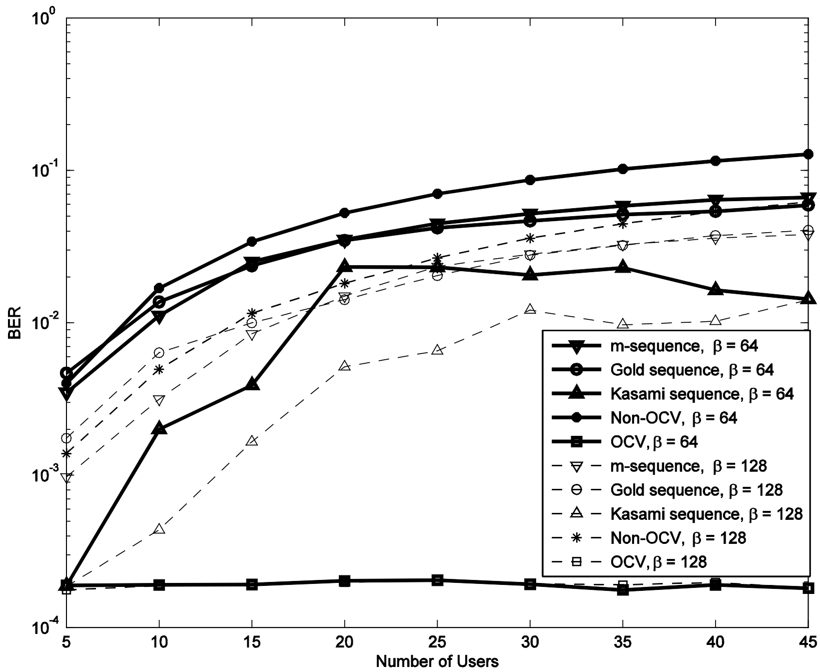


Fig. 3.11 Bit Error Rate versus number of users for different types of spreading codes for bit energy to noise ratio, E_b/N_o = 8 dB

In Fig. 3.10 and Fig. 3.11 the BER performance of the spread spectrum communication using conventional spreading codes and chaotic spreading codes are depicted. From Fig. 3.10(a-d) which depicts the BER curves for four user multi user communication system with different spreading factor, β varying from 16 to 256. As spread factor is increased the BER values for the system using m-sequence, gold codes and non-OCV converge whereas the system using OCV, OVSF and kasami codes converge with 1dB better than the previous case.

In Fig. 3.11 the BER performance of the multi user communication system is analysed with respect to the number of simultaneous users when different types of spreading codes are used to spread the information bit for two different values of spread factor, β (64 and 128). From Fig. 3.11 we see that for spread factor, β equal to 64 the BER performance of multi user system using m-sequence, gold codes and non-OCV are same for small number of users. For spread factor, β equal to 128 the BER performance of the system using m-sequence, gold codes and non-OCV are almost same whereas the system using kasami codes will be having better BER performance when compared to m-sequence, gold codes and Non-OCV because of its good cross correlation property. Whereas the BER performance of the system using OCV will be independent of the number of users which is evident from the fact that the use of orthogonal spreading codes does not cause MUI.

3.6 Summary

- ✓ Multi user chaotic communication system using Orthogonal Chaotic Vectors with coherent receiver is simulated and analyzed over AWGN. Through simulation it is observed that the performance of the multi user chaotic communication system using OCV will have better BER performance in comparison to conventional spread spectrum communication systems and chaotic communication systems using non OCV.
- ✓ It is very well known that using orthogonal vectors for multi user communication with perfectly synchronized receiver will have better performance due to the elimination of MAI. Thus, the results obtained for multi user chaotic communication using OCV with coherent receivers can be used as a benchmark results to compare other multi user chaotic communication system.

4. Multi user chaotic communication system using orthogonal chaotic vectors with coherent receivers over multi path fading channels

In this chapter the bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors over different multi path fading channels. In this chapter four different scenarios are considered namely, 1. Flat fading channel 2. Frequency selective fading channel 3. Time varying frequency selective fading channel with rake receiver 4. Fast frequency selective fading channel with rake receiver. The bit error rates for all these four cases are analyzed through simulation. Analytical expressions of bit error rate for flat fading channel case are derived using conventional Gaussian approximation method.

4.1 Bit Error Rate Analysis Over Flat Fading Channels

In this Section the bit error rate performance of multi user chaotic communication system using OCV over fading channels is discussed. It is assumed that the channel is slow fading. Fig.4.1 describes the fading channel with noise.

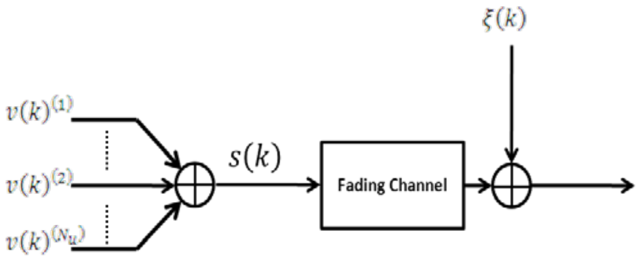


Fig. 4.1 Fading channel with noise

Transmitted signal for m^{th} data bit is given by,

$$s(k)_m = \sum_{i=1}^{Nu} d_m^{(i)} \hat{x}(k)_m^{(i)} \tag{4.1}$$

Received signal for the m^{th} transmitted data bit is given by,

$$r(k)_m = \alpha\, s(k)_m + \xi(k) \tag{4.2}$$

Where, α is the complex random number with Rayleigh or Ricean distribution with mean value E[α]. For all simulation and analysis purpose the mean value of magnitude of α, $E[|\alpha|^2]$ is assumed to be equal to unity. *ξ(k)* is complex gaussian random number with zero mean and variance equal to $N_o/2$

Correlator output for the m^{th} transmitted data bit of j^{th} is given by,

$$z_m^{(j)} = D + MUI + \xi$$

$$D = d_m^{(j)} \alpha\alpha^* \sum_{k=1}^{\beta} \left[\hat{x}(k)_m^{(j)} \right]^2 \quad MUI = d_m^{(j)} \alpha\alpha^* \sum_{i=1, i\neq j}^{N_u} \sum_{k=1}^{\beta} \left(\hat{x}(k)_m^{(i)} \hat{x}(k)_m^{(j)} \right) \quad \xi = \alpha^* \sum_{k=1}^{\beta} \xi(k) \hat{x}(k)_m^{(j)}$$

To simplify the representation the subscript 'm' can be neglected in further equations. Where, term D is the desired term which has the information bit, MUI is the multi user interference caused by the other users and ξ is the noise term.

Assuming, that $z^{(j)} \big| \left(d^{(j)} = -1 \right)$ and $z^{(j)} \big| \left(d^{(j)} = +1 \right)$ are random variables. According to central limit theorem for sum of large set of these random variables leads to Gaussian distribution. Therefore, BER for j^{th} user can be written as

$$BER^{(j)} = \frac{1}{2} erfc \left(\frac{E\left(z^{(j)} | d^{(j)} = +1 \right)}{\left(2 \operatorname{var}\left(z^{(j)} | d^{(j)} = +1 \right) \right)^{1/2}} \right) = \frac{1}{2} erfc \left(\frac{-E\left(z^{(j)} | d^{(j)} = -1 \right)}{\left(2 \operatorname{var}\left(z^{(j)} | d^{(j)} = -1 \right) \right)^{1/2}} \right) \tag{4.3}$$

Where, mean value of $\left(z^{(j)} | d^{(j)} = +1 \right)$ is given by,

$$E\left(z^{(j)} | d^{(j)} = +1 \right) = \beta |\alpha|^2 E\left[\left(\hat{x}(k)^{(j)} \right)^2 \right] \tag{4.4}$$

And variance is given by,

$$\operatorname{var}\left(z^{(j)} | d^{(j)} = +1 \right) = |\alpha|^4 \operatorname{var}\left[\sum_{k=1}^{\beta} \left[\hat{x}(k)^{(j)} \right]^2 \right] + |\alpha|^4 \operatorname{var}\left[\sum_{i=1, i\neq j}^{N_u} \sum_{k=1}^{\beta} \left(\hat{x}(k)^{(i)} \hat{x}(k)^{(j)} \right) \right]$$
$$+ \beta |\alpha|^2 E\left[\xi(k)^2 \right] E\left[\left(\hat{x}(k)^{(j)} \right)^2 \right] \tag{4.5}$$

Where,

$$E\left[\hat{x}(k)^{(j)} \right] = 0 \tag{4.6}$$

$$E\left[\left(\hat{x}(k)^{(j)} \right)^2 \right] = \frac{E_b}{\beta} = P_s \tag{4.7}$$

$$\operatorname{var}\left[\hat{x}(k)^{(j)} \right] = \frac{E_b}{\beta} \tag{4.8}$$

$$\operatorname{var}\left[\sum_{k=1}^{\beta} \left[\hat{x}(k)^{(j)} \right]^2 \right] = 0 \tag{4.9}$$

$$\operatorname{var}\left[\sum_{i=1, i\neq j}^{N_u} \sum_{k=1}^{\beta} \left(\hat{x}(k)^{(i)} \hat{x}(k)^{(j)} \right) \right] = \begin{cases} 0 & for \;\; \beta > N_u \\ \beta (N_u - 1) \mathrm{E}\left[\left(\hat{x}(k)^{(i)} \right)^2 \right] \mathrm{E}\left[\left(\hat{x}(k)^{(j)} \right)^2 \right] & for \;\; \beta \leq N_u \end{cases} \tag{4.10}$$

$$\operatorname{var}\left[\xi(k) \right] = E\left[\xi(k)^2 \right] = \frac{N_o}{2} \tag{4.11}$$

Where, P_s is average power transmitted by each user per bit and E_b is energy per bit

Using the equations from (4.1.4) to (4.1.11) in (4.1.3), we get

$$BER^{(j)} = \begin{cases} 0.5\,erfc\left[\left(\dfrac{2(N_u-1)}{\beta} + \left(\dfrac{|\alpha|^2 E_b}{N_o}\right)^{-1}\right)^{-\frac{1}{2}}\right] & for\ \ \beta \le N_u \\ \dfrac{1}{2} erfc\left(\dfrac{|\alpha|^2 E_b}{N_o}\right)^{\frac{1}{2}} \quad for\ \ \beta > N_u \end{cases} \tag{4.12}$$

$$BER_{\alpha}^{(j)}(\gamma) = \begin{cases} 0.5\,erfc\left[\left(\dfrac{2(N_u-1)}{\beta} + (\gamma)^{-1}\right)^{-\frac{1}{2}}\right] & for\ \ \beta \le N_u \\ \dfrac{1}{2} erfc(\gamma)^{\frac{1}{2}} \quad for\ \ \beta > N_u \end{cases} \tag{4.13}$$

Where, $\gamma = \dfrac{|\alpha|^2 E_b}{N_o}$ the received instantaneous signal to noise ratio.

4.1.1 Rayleigh fading channel

When, α is Rayleigh-distributed γ (the received instantaneous signal to noise ratio per bit) will be chi-square distributed and has the form [49],

$$f_{Rayliegh} = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}, \gamma \ge 0 \tag{4.14}$$

where, $\bar{\gamma} = E[\gamma] = E\left[|\alpha|^2\right] \bullet \dfrac{E_b}{N_o}$

Therefore, the average BER for j^{th} user is

$$BER_{Rayleigh}^{(j)} = \int_0^{\infty} BER_{\alpha}^{(j)}(\gamma)\, f_{Rayleigh}(\gamma)\ d\gamma \tag{4.15}$$

In Fig. 4.2 both simulated and theoretical bit error rates of the multi user chaotic communication system using OCV is shown. From Fig. 4.2 it is evident that the theoretical bit error rates calculated using equation (4.15) closely matches with that of simulated bit error rates.

In Fig. 4.3. and Fig. 4.4. the simulated and theoretical bit error rates are plotted as a function of spread factor, β. As spread factor, β, is increased the bit error rate initially decreases till β < Number of users, N_u, and reaches minimum value at $\beta = N_u$. In Fig. 4.3 the simulated bit error rates for multi user chaotic communication system using orthogonal chaotic vectors are compared with its non-orthogonal counterpart. It is observed that when spread factor, β is increased above N_u the bit error rate of the multi user chaotic communication system using orthogonal chaotic vectors slowly increases and converges to the bit error rates of non-orthogonal chaotic vector based multi user chaotic communication system. In Fig. 4.4. the simulated bit error rates are compared with that of theoretical bit error rates computed using equation (4.15).

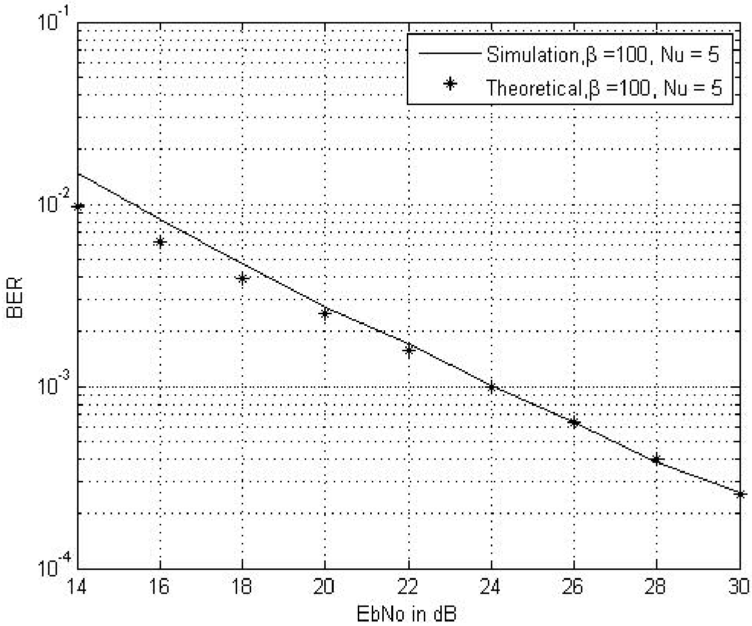


Fig. 4.2 Simulated and theoretical BERs versus Eb/No, Over Rayleigh fading channel

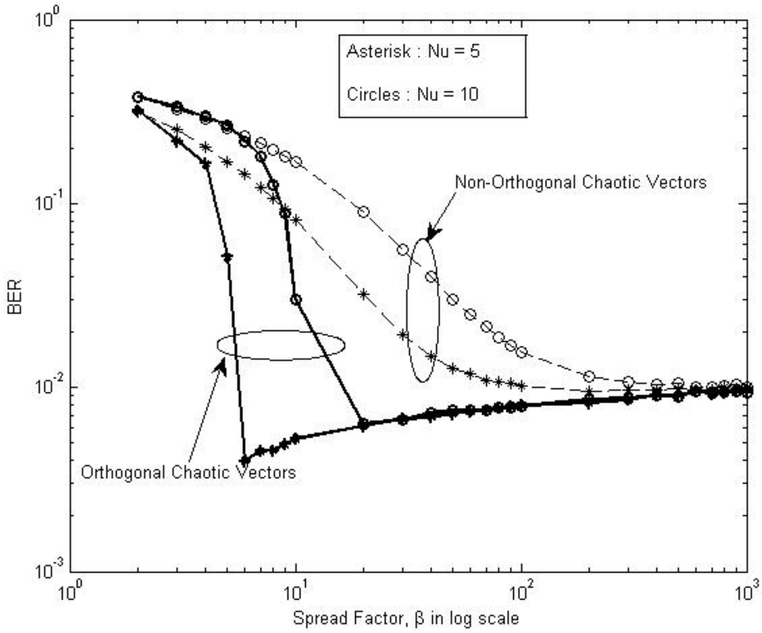


Fig. 4.3Simulated Bit Error rates v/s Spread Factor, β, for E_b/N_o = 15 dB

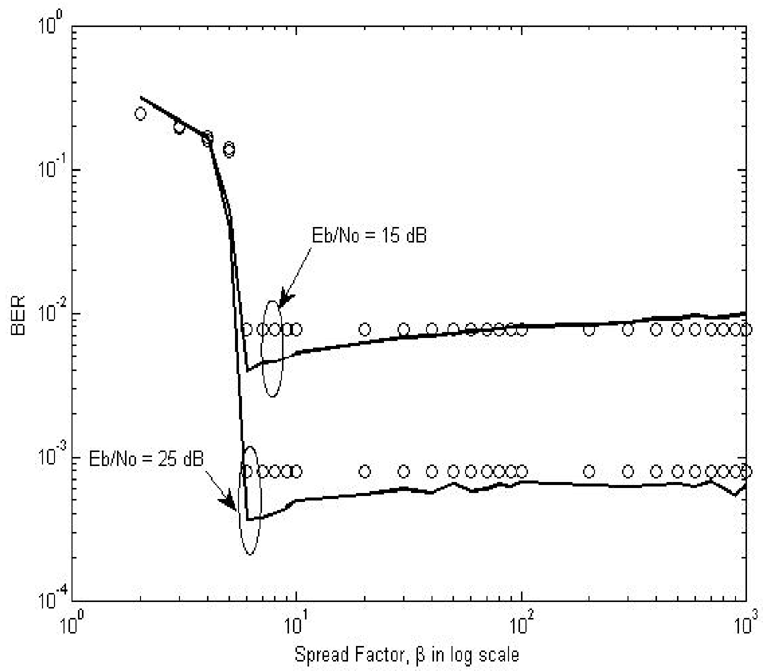


Fig. 4.4Simulated and Theoretical Bit error rates v/s Spread Factor, β for Number of Users, N_u = 5
Solid Line : Simulated BER, Circles : Theoretical BER

Fig. 4.5. gives the comparison of bit error rates of multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors as a function of number of users, N_u. Theoretical Bit error rates calculated using equation (4.15) for comparison.

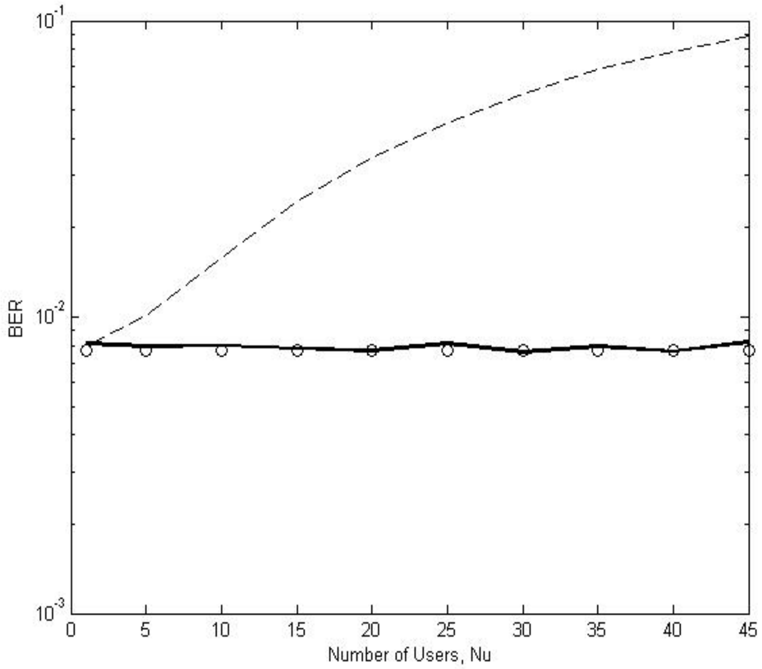


Fig. 4.5 Simulated and Theoretical BER v/s Number Of Users, N_u for spread factor, β = 100, E_b/N_o = 15 dB, Solid Line : Simulated BER for OCV , Dashed Line : Simulated BER for Non-OCV, Circles : Theoretical BER

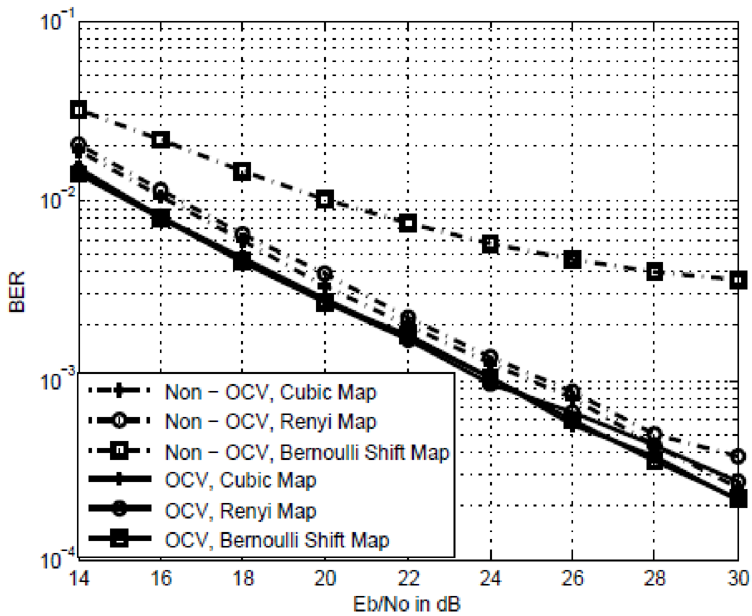


Fig. 4.6 Comparison of BERs v/s Eb/No plots for number of users = 5 and spread factor, β = 100

In Fig. 4.6 the simulated bit error rates for multi user chaotic communication system when chaotic sequences generated from different chaotic maps are compared. When ortho-normalized the performance of the multi user chaotic communication system will be independent of the chaotic map used to generate the chaotic sequences whereas in the other case the bit error rate performance of the multi user chaotic communication system depends on the variance of the cross correlation co-efficient which is different for different chaotic map.

4.1.2 Ricean fading channel

If the received signal has a dominant component then channel is considered to be Ricean fading channel. The conventional single user DCSK BER performance over multipath Ricean fading channel has been investigated in [36], where the Ricean distribution with a PDF is derived as

$$f_{Ricean} = \frac{(1+K)e^{-K}}{\gamma'} \exp\left(-(1+K)\frac{\gamma}{\gamma'}\right) G(\gamma) \tag{4.16}$$

Where $G(\gamma) = I_o\left[\sqrt{\frac{4K(1+K)\gamma}{\gamma'}}\right]$ and $I_o = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{x\cos\theta}\, d\theta$ is the modified Bessel function of the first kind and zero order, $K = \frac{A^2}{2\sigma^2}$ is the Ricean factor, *A* denotes the amplitude of the dominant stationary signal, σ^2 is the average power of the scattered signals. Therefore, the average BER for j^{th} user is

$$BER_{Ricean} = \int_0^{\infty} BER_{\alpha}(\gamma)\, f_{Ricean}(\gamma)\, d\gamma \tag{4.17}$$

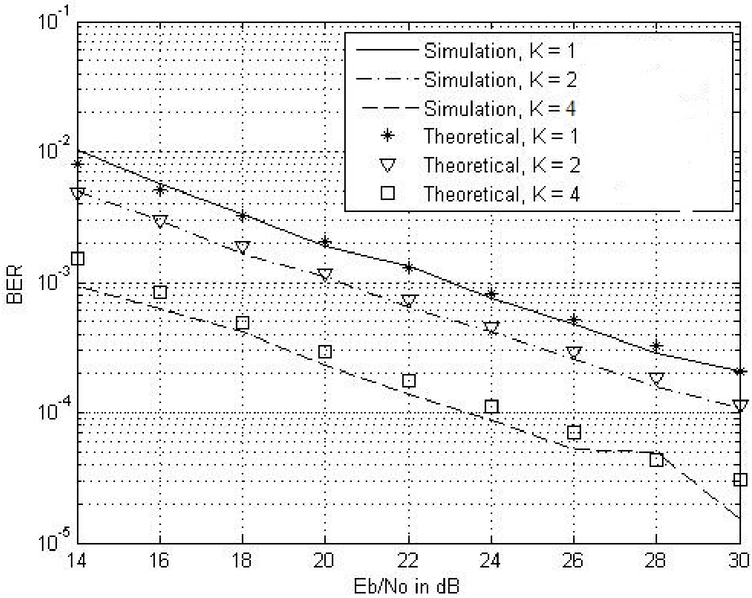


Fig. 4.7 Simulated and theoretical BERs versus Eb/No, Over Ricean fading channel for different values of K, β = 100 and Number of users = 5

In Fig. 4.7 both simulated and theoretical bit error rates of the multi user chaotic communication system using orthogonal chaotic vectors are shown for different values of Ricean factor, K. From Fig. 4.7 it is evident that the theoretical bit error rates calculated using equation (4.17) closely matches with that of simulated bit error rates. In Fig. 4.8. and Fig. 4.9. the simulated and theoretical bit error rates are plotted as a function of spread factor, β. As spread factor, β, is increased the bit error rate initially decreases till $\beta <$ Number of users, N_u, and reaches minimum value at $\beta = N_u$.

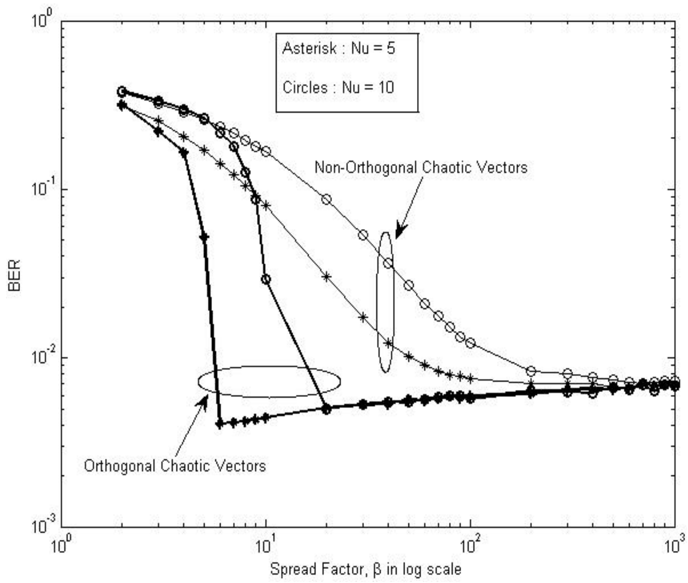


Fig. 4.8Simulated Bit Error Rates v/s Spread Factor, β for Eb/No = 15 dB

In Fig. 4.8 the simulated bit error rates for multi user chaotic communication system using orthogonal chaotic vectors are compared with its non-orthogonal counterpart. It is observed that when spread factor, β is increased above N_u the bit error rate of the multi user chaotic communication system using orthogonal chaotic vectors slowly increases and converges to the bit error rates of non-orthogonal chaotic vector based multi user chaotic communication system. In Fig. 4.9. the simulated bit error rates are compared with that of theoretical bit error rates computed using equation (4.17).

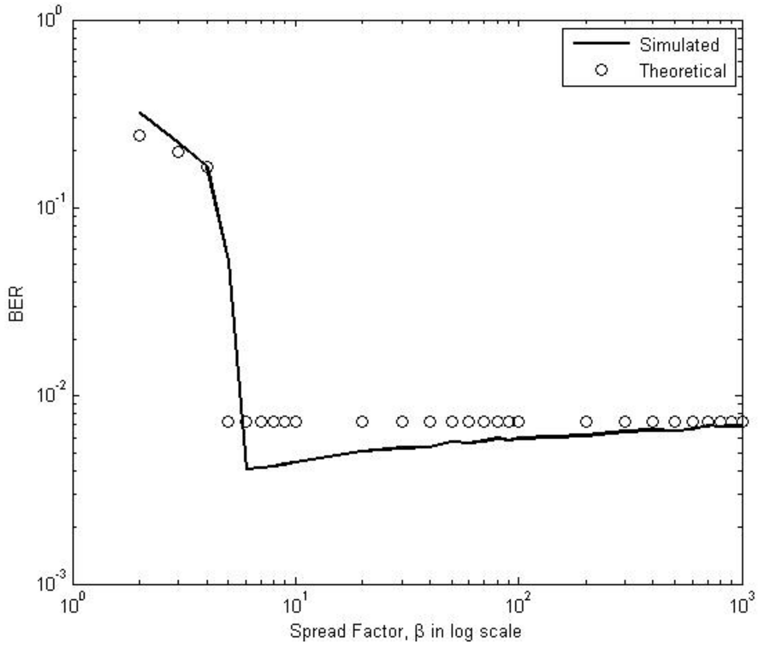


Fig. 4.9Simulated and Theoretical Bit Error Rates v/s Spread Factor, β for Eb/No = 15 dB, Solid Line : Simulated BER, Circles : Theoretical BER

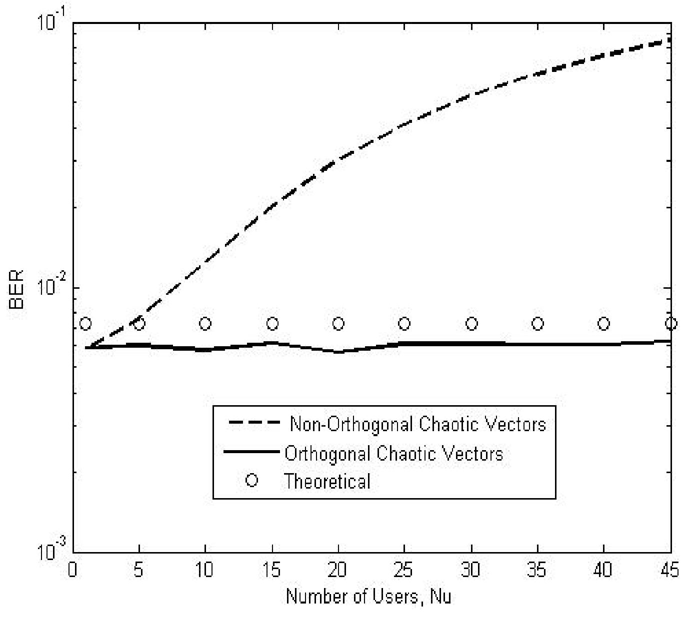


Fig. 4.10 Simulated and Theoretical Bit Error Rates v/s Number Of Users, N_u for Eb/No = 15 dB

Fig. 4.10. gives the comparison of bit error rates of multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors as a function of number of users, N_u. Theoretical Bit error rates calculated using equation (4.17) for comparison.

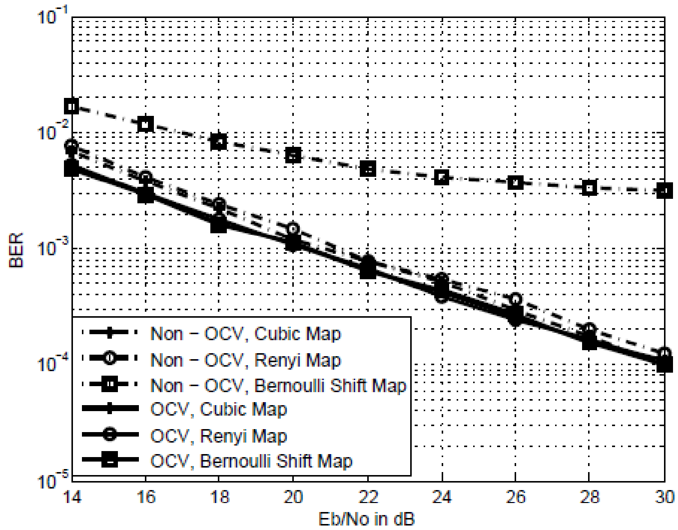


Fig. 4.11Comparison of BERs v/s Eb/No plots for number of users = 5, β = 100 and K = 2

In Fig. 4.11 the simulated bit error rates for multi user chaotic communication system when chaotic sequences generated from different chaotic maps are compared. When ortho-normalized the performance of the multi user chaotic communication system will be independent of the chaotic map used to generate the chaotic sequences whereas in the other case the bit error rate performance of the multi user chaotic communication system depends on the variance of the cross correlation co-efficient which is different for different chaotic map.

4.2 Bit Error Rate Analysis over Frequency Selective Fading Channel

In this section the bit error rate performance of multi user chaotic communication system using orthogonal chaotic vectors over frequency selective fading channel are analyzed through simulation. The channel model presented in [50] is taken as reference for simulations.

4.2.1. Channel Model

The channel impulse response is given by [50],

$$h(t) = 0.642\delta(t) + 0.603\delta(t-\tau_1) + 0.4264\delta(t-\tau_2) \tag{4.18}$$

$$h(n) = 0.642\delta(n) + 0.603\delta(n-1) + 0.4264\delta(n-2) \tag{4.19}$$

Where, δ(t) and δ(n) are impulse function in continuous and discrete time domain.

Fig. 4.12. Shows the pole-zero plot and frequency response of the channel impulse response given by equation (4.18). From Fig. 4.12(b) it is clear that the channel impulse response has deep fade and linear phase response.

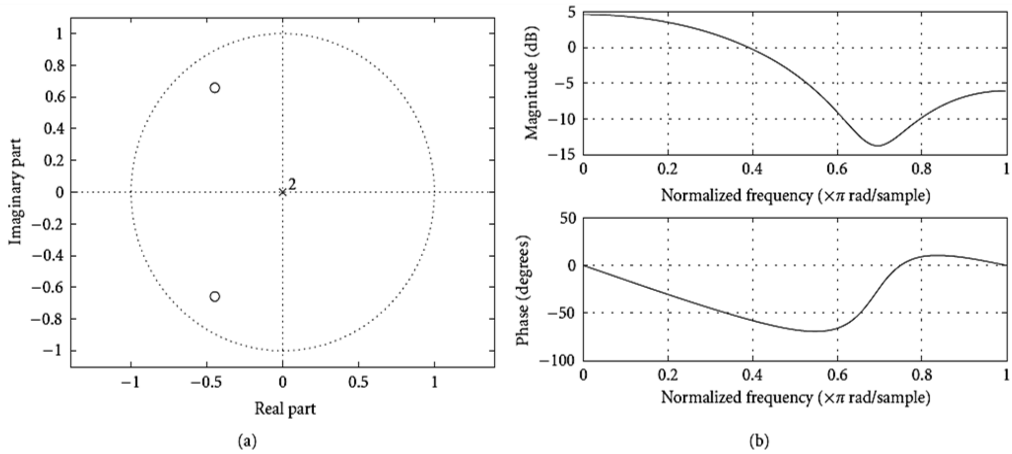


Fig. 4.12. (a) Zeros of $H(z)$; (b) amplitude and phase responses of the channel[44].

Received Signal for the m^{th} transmitted information bit is given by,

$$r_m(k) = s_m(k) * h(k) + \xi(k) \tag{4.20}$$

$$r_m(k) = \sum_{n=0}^{N_p-1} a_n s_m(k-n) + \xi(k)$$

Where, $s_m(k) = \sum_{i=1}^{Nu} d_m^{(i)} \hat{x}(k)_m^{(i)}$ is the transmitted signal and a_n is the channel co-efficient of n^{th} path.

Correlator output, $z_m^{(j)}$ of the j^{th} user for m^{th} transmitted information bit is given by,

$$z_m^{(j)} = \sum_{k=1}^{\beta} r_m(k)\hat{x}_m^{(j)}(k)$$

$$\begin{aligned} z_m^{(j)} = {} & a_0 \sum_{k=1}^{\beta} d_m^{(j)} \left[\hat{x}_m^{(j)}(k)\right]^2 + a_0 \sum_{k=1}^{\beta} \sum_{i=1,\, j\neq i}^{N_u} d_m^{(i)} \hat{x}_m^{(i)}(k)\hat{x}_m^{(j)}(k) + \sum_{n=1}^{N_p-1} \sum_{k=n+1}^{\beta} \sum_{i=1}^{N_u} a_n d_m^{(i)} \hat{x}_m^{(i)}(k-n)\hat{x}_m^{(j)}(k) + \\ & \sum_{n=1}^{N_p-1} \sum_{k=1}^{n} \sum_{i=1}^{N_u} a_m d_{m-1}^{(i)} \hat{x}_{m-1}^{(i)}(\beta-n+k)\hat{x}_m^{(j)}(k) + \sum_{k=1}^{\beta} \xi_m(k)\hat{x}_m^{(j)}(k) \end{aligned} \tag{4.21}$$

$$z_m^{(j)} = D + MUI + Intra\ SI + Inter\ SI + \xi \tag{4.22}$$

In equation (4.22) *D* is the desired term which contains information bit, *MUI* is the multi user interference caused by other users, *Intra SI* is the Intra Symbol Interference caused by the partial auto and cross correlations, *Inter SI* is the Inter Symbol Interference caused by the previous information bit due to multipath nature of the channel. Since, the chaotic vectors are orthonormal to each other when spread factor,

β is greater than number of users, N_u the term *MUI* will be eliminated. Thus the factors affecting the bit error rate performance will be only Intra SI and Inter SI.

The values of the different parameters used for simulation are given below:

Chip Duration, $T_c = 0.8$ µsec

Sampling Duration, $T_s = 0.8$ µsec

Sampling Frequency, $F_s = \frac{1}{T_s} = 1.25$ MHz

Path Delay, $\tau_i = i * T_c$

RMS Delay Spread, $\sigma_\tau = 0.6$ µsec

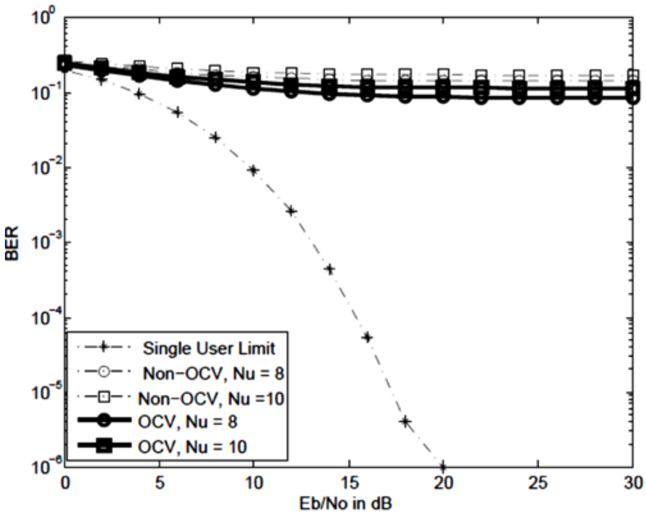
The channel parameters are chosen with respect to CDMA2000 1X standard [51]. The parameters for CDMA 2000 1X is given in the table given below,

Table 4.1 CDMA 2000 1X Parameters **[51]**

Bandwidth	1.25 MHz
Chip Rate, T_c	1.2288 Mcps
Data Rate, R	9.6 kbps

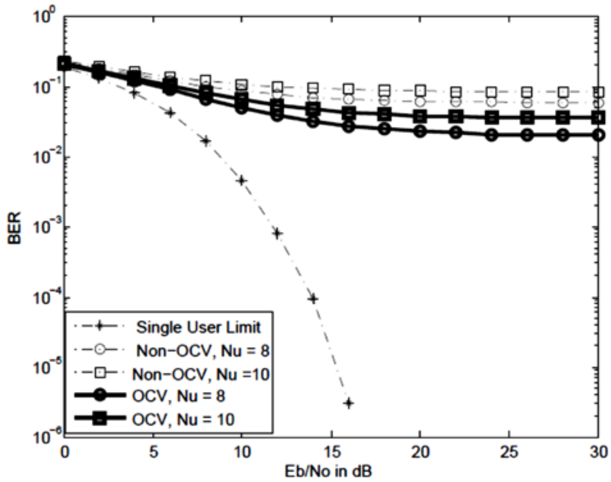
4.2.2. Simulation Results

Bit error rate performance of the multiuser chaotic communication system over frequency selective channel is simulated for different values of spread factor β and number of users, Nu.

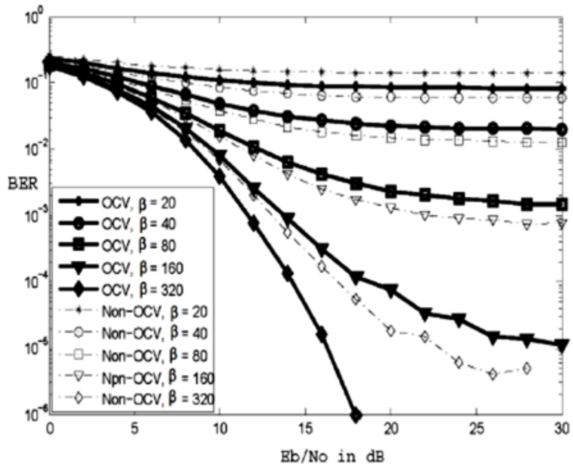


(a) Spread Factor, $\beta = 20$

Fig. 4.13(a-c). BER Performance of Multi user Chaotic Communication system over frequency selective fading channel for different values of spread factor, β and number of users Nu



(b) Spread Factor, β = 40



(c) Number Of Users, Nu = 8

Fig. 4.13(a-c). BER Performance of Multi user Chaotic Communication system over frequency selective fading channel for different values of spread factor, β and number of users Nu (Continued)

The simulation results are shown in Fig. 4.13(a-b). In Fig. 4.13 (a-b) the simulated bit error rates are plotted for different values of number of users, N_u (8 and 10) for fixed values of spread factor, β. In each plot from Fig. 4.13(b) to 4.13(b) the single user bit error rate for single user case under frequency selective fading channel is plotted for comparison. Fig. 4.13(c) compares the simulated bit error rates with respect to bit energy to noise ratio (E_b/N_o) for different values of spread factor, β with fixed value of number of users, N_u.

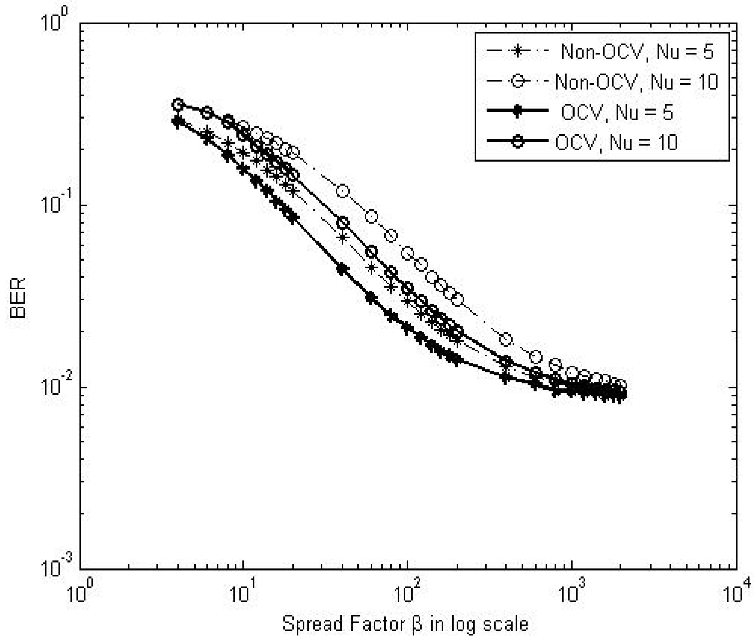


Fig. 4.14 BER v/s Spread Factor β for Number of Users Nu = 5 and 10 and Eb/No ratio 8 dB

Fig. 4.14 compares the bit error rate performance of the multi user chaotic communication system over frequency selective fading channel as a function of spread factor, β with constant E_b/N_o ratio. As spread factor, β is increased the bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors will converge to its non-orthogonal counterpart.

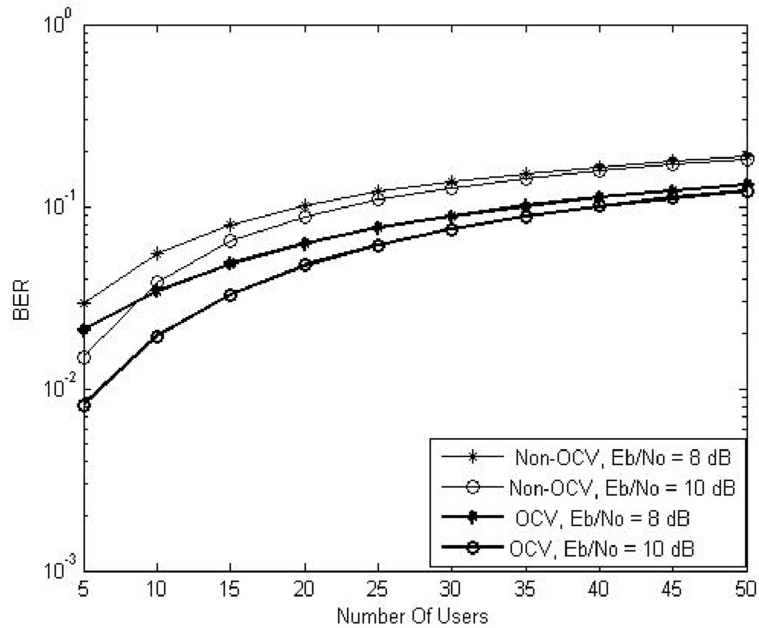


Fig. 4.15 BER v/s Number Of Users, Nu plots for Eb/No ratio 8 dB and 10 dB

In Fig. 4.15 the simulated bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors are plotted as a function of number of users, N_u.

4.3 Bit Error Rate Analysis over Time Varying Frequency Selective Fading Channel with Rake Receiver

In this section the bit error rate performance of multi user chaotic communication system using orthogonal chaotic vectors over time varying frequency selective fading channel with rake receiver are analyzed through simulation. It is assumed that the rake receiver used is perfect i.e., the channel coefficients are known at the receiver and number of rake fingers is equal to the number of multi path.

4.3.1. Channel Model

In this type of channel the channel coefficients vary with respect to time. The channel impulse response of the time varying frequency selective fading channel is given by,

$$h(t,\tau') = \sum_{n=0}^{Np-1} a_n(\tau')\delta(t-\tau_n) = \sum_{n=0}^{Np-1} a_n(\tau')\delta(t-nT_c) \tag{4.23}$$

$$h(k,\tau') = \sum_{n=0}^{Np-1} a_n(\tau')\delta(k-n) \tag{4.24}$$

Where, δ(t) and δ(n) represents the impulse function in continuous and discrete time domain, $a_n(\tau')$ represents the channel co-efficient for the nth path in multi path channel which is varying with time, τ'.

Received Vector: $r(k) = \sum_{n=0}^{N_p-1} a_n S(k-n) + \xi(k)$

Output of l^{th} rake finger for p^{th} user and j^{th} transmitted data bit is given by,

$$\begin{aligned} rake(l)_j^{(p)} = {} & \sum_{k=l+1}^{\beta+l} a_{l,j} a_{l,j}^{*} d_j^{(p)} \left[\hat{x}_j^{(p)}(k)\right]^2 + \sum_{q=1,p\neq q}^{N_u} \sum_{k=l+1}^{\beta+l} d_j^{(p)} a_{l,j} a_{l,j}^{*} \hat{x}_j^{(p)}(k)\hat{x}_j^{(q)}(k) + \\ & \sum_{m=0,l\neq m}^{N_p-1} \sum_{q=1}^{N_u} \sum_{k=m+1}^{\beta} d_j^{(p)} a_{l,j} a_{l,j}^{*} \hat{x}_j^{(p)}(k)\hat{x}_j^{(q)}(k-m) + \sum_{m=l}^{N_p-1} \sum_{q=1}^{N_u} \sum_{k=1}^{m+1} d_{j-1}^{(p)} a_{m,j-1} a_{l,j}^{*} \hat{x}_{j-1}^{(p)}(\beta-m-1+k)\hat{x}_j^{(p)}(k) + \\ & \sum_{m=0}^{l} \sum_{q=1}^{N_u} \sum_{k=\beta-m}^{\beta} d_{j+1}^{(p)} a_{m,j+1} a_{l,j}^{*} \hat{x}_{j+1}^{(q)}(k-\beta+m+1)\hat{x}_j^{(p)}(k) + \sum_{k=1}^{\beta} \xi_j(k)\hat{x}_j^{(p)}(k) \end{aligned} \quad 4.25)$$

$$rake(l)_j^{(p)} = D + MUI + Intra\ SI + Inter\ SI1 + Inter\ SI2 + \xi \tag{4.26}$$

Decision Variable: $z_j^{(p)} = \sum_{l=0}^{N_p-1} rake(l)_j^{(p)}$

The term *D* in equation (4.26) is the desired term containing information bit, *MUI* is the multi user interference caused by multi user, *Inter SI1* and *Inter SI2* are the inter symbol interferences caused by the previous information bit and next transmitted information bit respectively and *Intra SI* is the Intra symbol interference caused by the partial auto and cross correlation of the transmitted signal.

The values of the different parameters used for simulation are as given below:

Chip Duration, $T_c = 0.8$ µsec

Sampling Duration, $T_s = 0.8$ µsec

Sampling Frequency, $F_s = \frac{1}{T_s} = 1.25$ MHz

Path Delay, $\tau_n = n * T_c$

Channel Co-efficients: a_n, Complex random number with rayleigh or Ricean distribution.

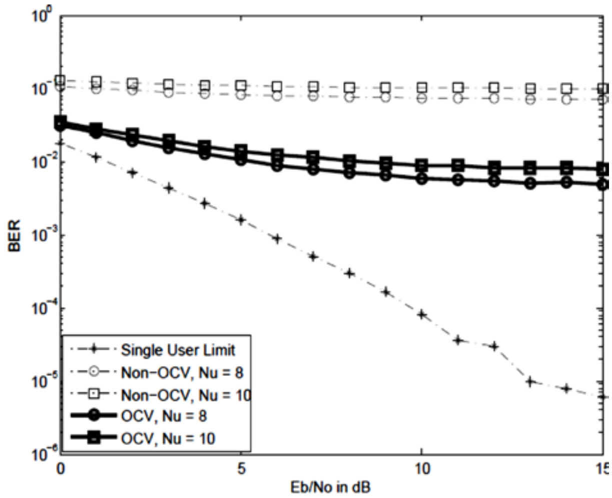
For simulation it is assumed that the delay of each path is equal to the integer multiple of chip rate and the channel coefficients are constant during the bit duration.

4.3.2. *Simulation Results*

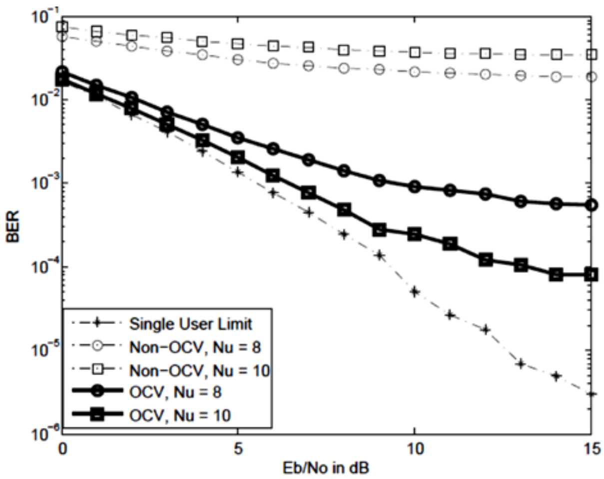
In this section the simulated bit error rates of the multi user chaotic communication system using orthogonal chaotic vectors over time varying multipath fading channel are presented and discussed. First the simulated bit error rates for the case when the channel coefficients are rayleigh distributed are presented followed by the case when channel coefficients are Ricean distributed.

BER performance when channel coefficients are Rayleigh distributed:

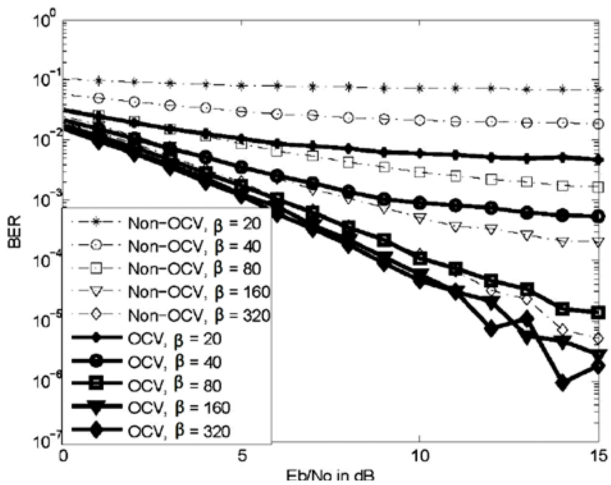
Bit error rate performance of the Multiuser chaotic communication system over time varying frequency selective channel with channel coefficients being rayleigh distributed is simulated for different values of spread factor β and number of users, Nu. And the simulation results are shown in Fig. 4.16(a-c) for the case of number of multipath, $N_p = 3$. From Fig. 4.16 (a-b) the simulated bit error rates are plotted for different values of Number of users, N_u (8 and 10) for fixed values of spread factor, β. In each plot from Fig. 4.16(a) to 4.16(b) the single user bit error rate for single user case under frequency selective fading channel is plotted for comparison. Fig. 4.16(c) compares the simulated bit error rates with respect to bit energy to noise ratio (E_b/N_o) for different values of spread factor, β, with fixed value of number of users, N_u. In Fig. 4.16(d – f) the simulated bit error rates are plotted for different number of multipath, N_p . As number of multi path increases so the number of rake fingers are increased since for all simulations it is assumed that the number of rake receivers is equal to the number of multpath.



(a) Spread Factor, β = 20, Number of multi path, N_p = 3

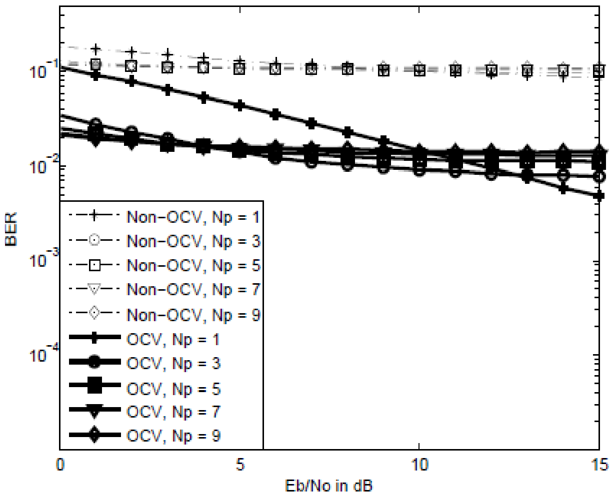


(b) Spread Factor, β = 40, Number of multi path, N_p = 3

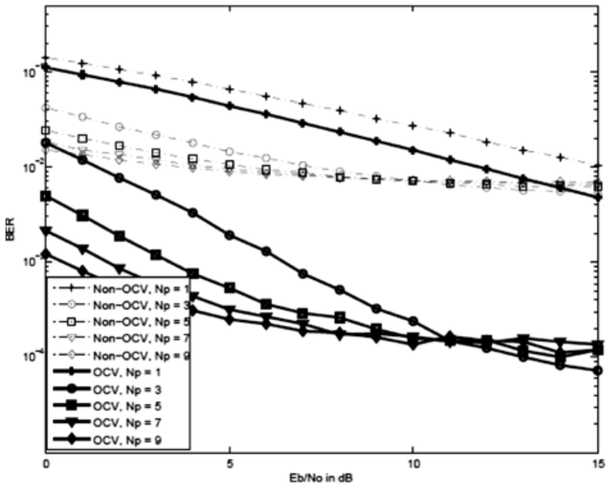


(c)Number of Users, Nu = 8, Number of multi path, N_p = 3

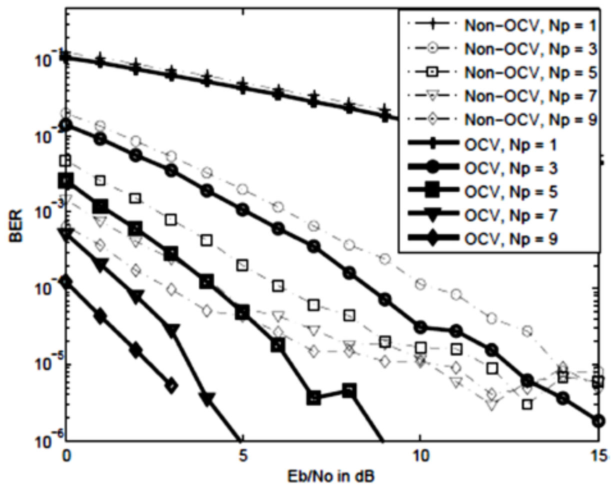
Fig. 4.16(a-f). BER Performance of multi user chaotic system over time varying rayleigh frequency selective channel for different values of spread factor, β and number of users, Nu (Continued)



(d) Spread Factor, β = 20, Number of Users, Nu = 10



(e) Spread Factor, β = 80, Number of Users, Nu = 10



(f) Spread Factor, β = 320, Number of Users, Nu = 10

Fig. 4.16(a-f). BER Performance of multi user chaotic system over time varying rayleigh frequency selective channel for different values of spread factor, β and number of users, Nu (Continued)

Fig. 4.17 compares the bit error rate performance of the multi user chaotic communication system over time varying rayleigh frequency selective fading channel as a function of spread factor, β with constant E_b/N_o ratio. As spread factor, β is increased the bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors decrease the orthogonal chaotic vectors better bit error rates than non-orthogonal chaotic vectors. In Fig. 4.18 the simulated bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors are plotted as a function of number of users, N_u.

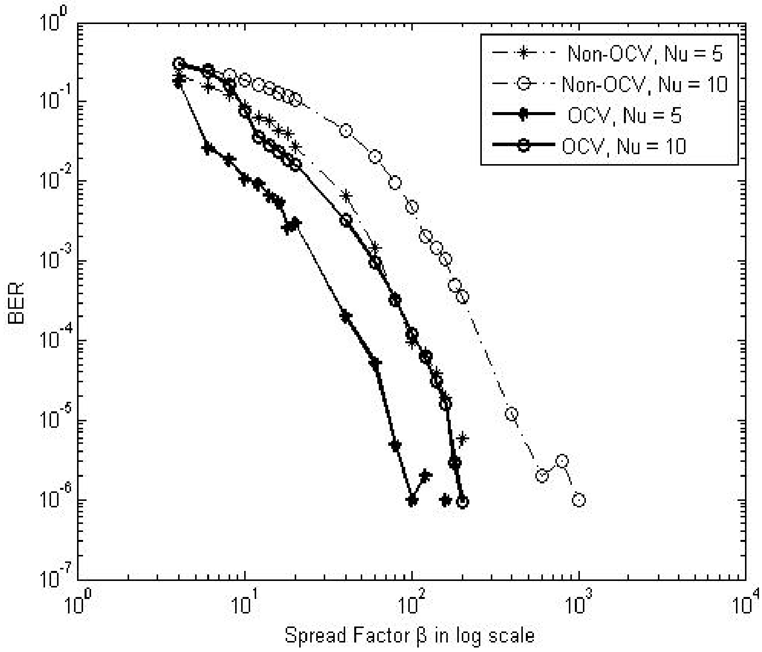


Fig. 4.17 BER v/s Spread Factor β for Number of Users Nu = 5 and 10 and Eb/No ratio 8 dB

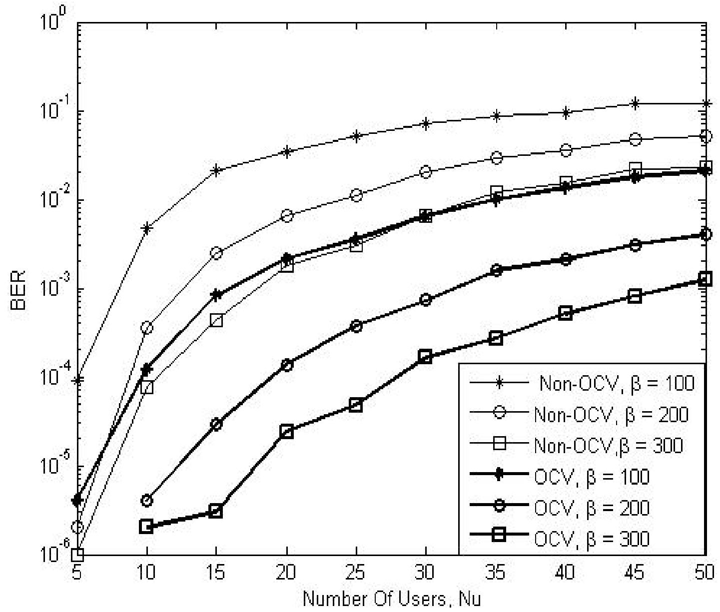


Fig. 4.18 BER v/s Number Of Users, Nu plots for Eb/No ratio 8 dB

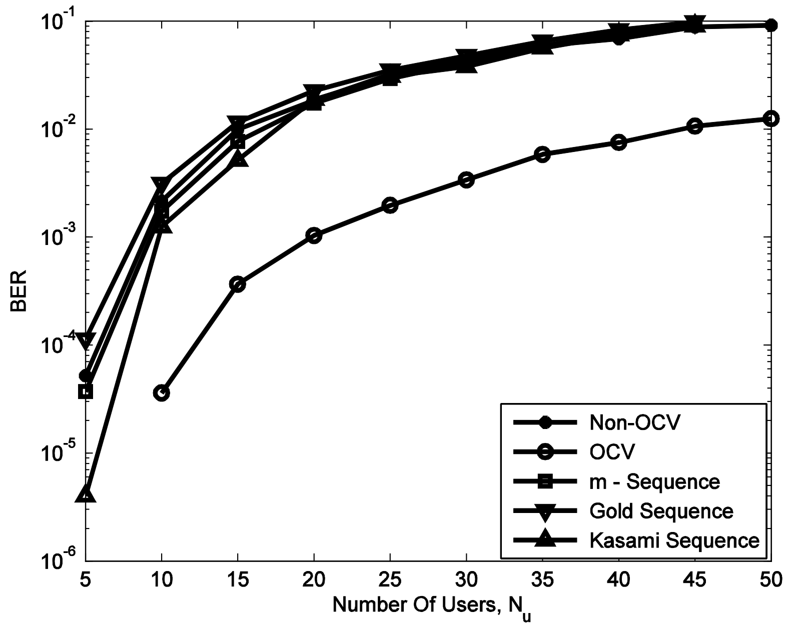
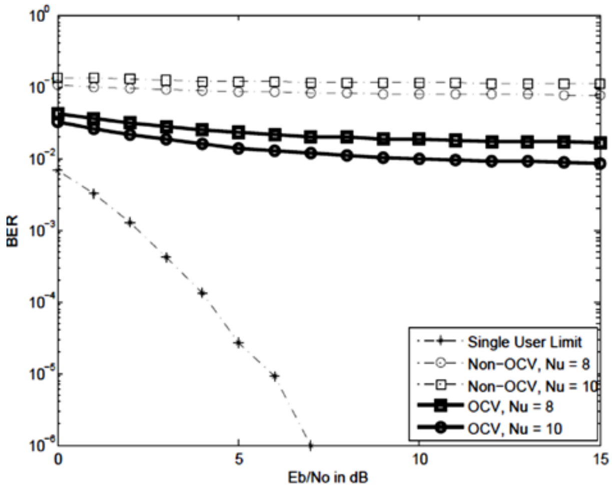


Fig. 4.19 Simulated Bit Error Rate versus Number of Users, N_u for spread Factor, β = 128 and E_b/N_o = 8 dB

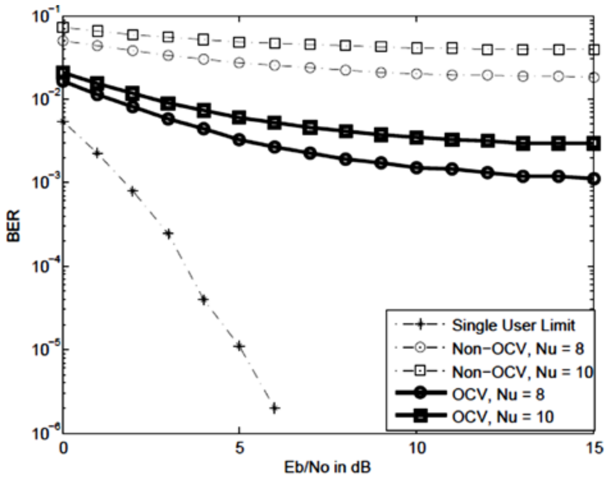
In Fig. 4.19 the simulated bit error rates of multi user chaotic communication using OCV and Non-OCV are compared with that of spread spectrum communication systems using conventional spreading sequences (m-sequence, Gold codes and Kasami Codes). From Fig. 4.19 it is clear that the bit error rate performance of the multi user chaotic communication system using OCV is better compared to non-OCV and conventional spreading sequences.

BER Performance when channel co-efficients are Ricean distributed:

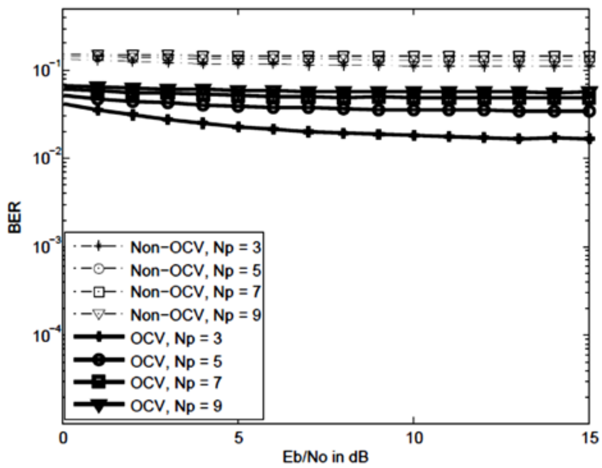
Bit error rate performance of the Multiuser chaotic communication system over time varying frequency selective channel with channel coefficients being rayleigh distributed is simulated for different values of spread factor β and number of users, Nu. And the simulation results are shown in Fig. 4.20(a-b) for the case of number of multipath, N_p = 3. From Fig. 4.20 (a-b) the simulated bit error rates are plotted for different values of Number of users, N_u (8 and 10) for fixed values of spread factor, β. In each plot from Fig. 4.20(a) to 4.20(b) the single user bit error rate for single user case under frequency selective fading channel is plotted for comparison. In Fig. 4.20(c – e) the simulated bit error rates are plotted for different number of multipath, N_p . As number of multi path increases so the number of rake fingers are increased since for all simulations it is assumed that the number of rake fingers is equal to the number of multpath.



(a) Spread Factor, β = 20, Number of Multipath, N_p=3

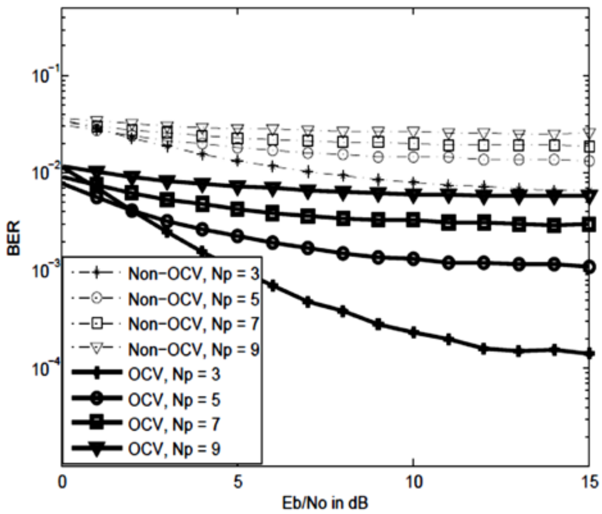


(b) Spread Factor, β = 40, Number of Multipath, N_p=3

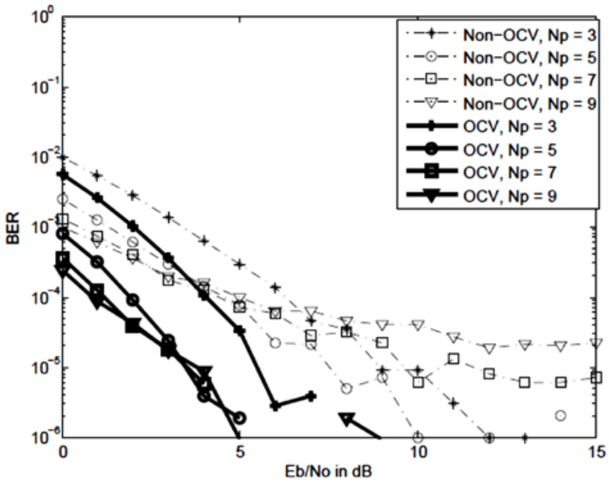


(c) Spread Factor, β = 20, Number Of Users, Nu = 10

Fig. 4.20(a-e). BER Performance of multi user chaotic system over time varying rayleigh frequency selective channel for different values of spread factor, β and number of users, Nu



(d) Spread Factor, β = 80, Number Of Users, Nu = 10



(e) Spread Factor, β = 320, Number Of Users, Nu = 10

Fig. 4.20(a-e). BER Performane Of Multi user chaotic communication system over time varying Ricean frequency selective channel for different values of spread factor, β and number of users, Nu (Continued)

Fig. 4.21 compares the bit error rate performance of the multi user chaotic communication system over time varying rayleigh frequency selective fading channel as a function of spread factor, β with constant E_b/N_o ratio. As spread factor, β is increased the bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors decrease the orthogonal chaotic vectors better bit error rates than non-orthogonal chaotic vectors. In Fig. 4.22 the simulated bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors are plotted as a function of number of users, N_u.

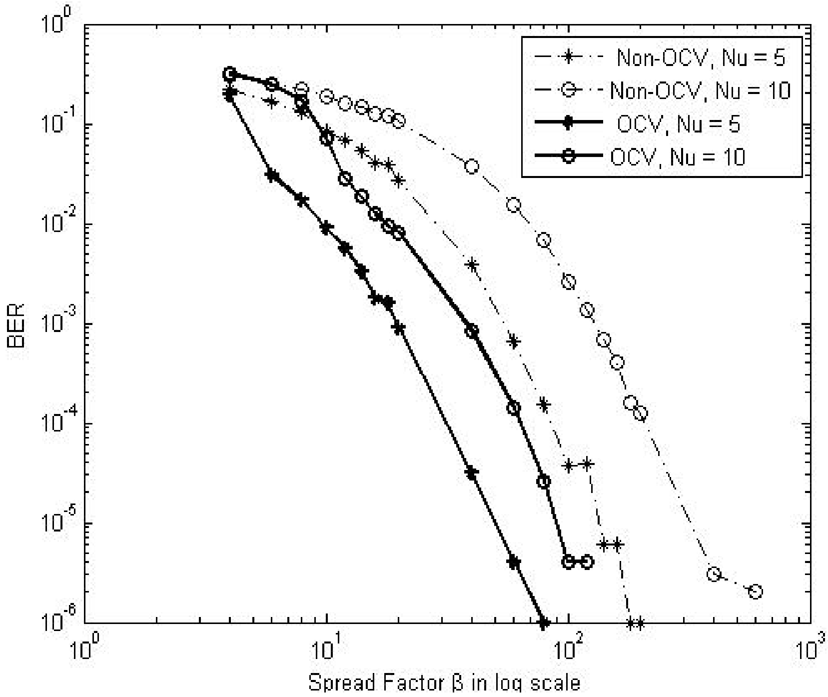


Fig. 4.21 BER v/s Spread Factor β for Number of Users Nu= 5 and 10, Number of Multipath, N_p=3

And Eb/No ratio 8 dB

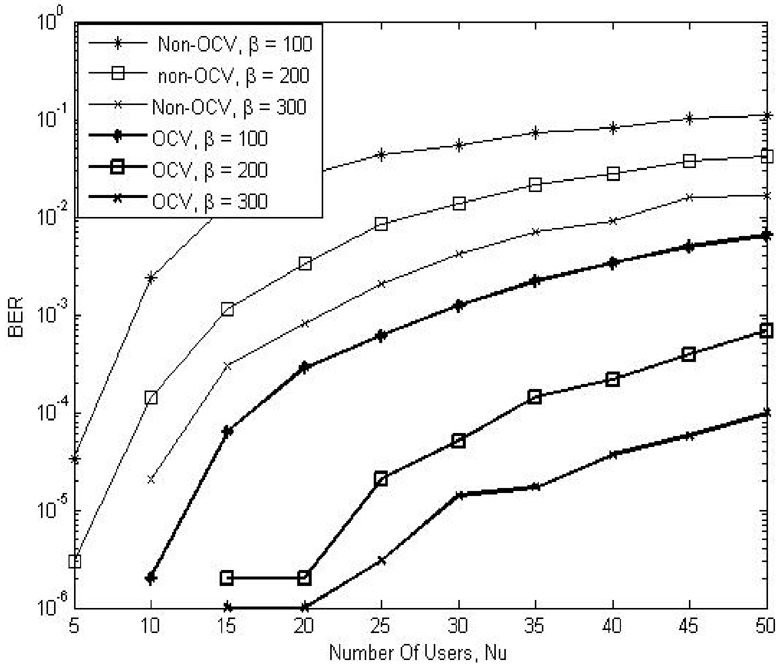


Fig. 4.22 BER v/s Number Of Users, Nu plots for Number of Multipath, N_p=3

And Eb/No ratio 8 dB

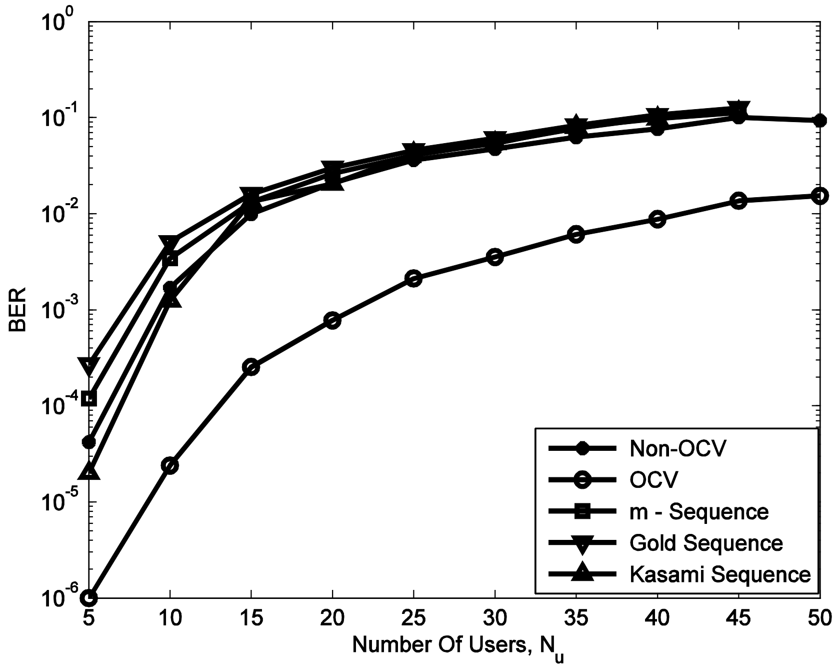


Fig. 4.23 Simulated Bit Error Rate versus Number of Users, N_u for Spread Factor, β =128 and E_b/N_o= 8dB

In Fig. 4.23 the simulated bit error rates of multi user chaotic communication using OCV and Non-OCV are compared with that of spread spectrum communication systems using conventional spreading sequences (m-sequence, Gold codes and Kasami Codes). From Fig. 4.23 it is clear that the bit error rate performance of the multi user chaotic communication system using OCV is better compared to non-OCV and conventional spreading sequences.

4.4 Bit Error Rate Analysis over Fast Frequency Selective Fading Channel with Rake Receiver

In this section the simulated bit error rates of the multi user chaotic communication system using orthogonal chaotic vectors over fast frequency selective fading channel are presented and discussed. First the simulated bit error rates for the case when the channel coefficients are Rayleigh distributed are presented followed by the case when channel coefficients are Ricean distributed.

4.4.1. Channel Model

In this type of channel the channel coefficients vary with respect to time. The channel impulse response of the time varying frequency selective fading channel is given by,

$$h(t,\tau')=\sum_{n=0}^{Np-1}a_n(\tau')\delta(t-\tau_n)=\sum_{n=0}^{Np-1}a_n(\tau')\delta(t-nT_c) \qquad (4.27)$$

$$h(k,\tau') = \sum_{n=0}^{Np-1} a_n(\tau')\,\delta(k-n) \tag{4.28}$$

Where, δ(t) and δ(n) represents the impulse function in continuous and discrete time domain, $a_n(\tau')$ represents the channel co-efficient for the nth path in multi path channel which is varying with time, τ'.

Received Vector: $r(k) = \sum_{n=0}^{N_p-1} a_n S(k-n) + \xi(k)$

Output of l^{th} rake finger of the j^{th} user for p^{th} transmitted data bit is given by:

$$\begin{aligned} rake(l)_j^{(p)} = {} & \sum_{k=l+1}^{\beta+l} a_{l,j} a_{l,j}^{*} d_j^{(p)} \left[\hat{x}_j^{(p)}(k)\right]^2 + \sum_{q=1,\, p\neq q}^{N_u} \sum_{k=l+1}^{\beta+l} d_j^{(p)} a_{l,j} a_{l,j}^{*} \hat{x}_j^{(p)}(k)\,\hat{x}_j^{(q)}(k) + \\ & \sum_{m=0, l\neq m}^{N_p-1} \sum_{q=1}^{N_u} \sum_{k=m+1}^{\beta} d_j^{(p)} a_{l,j} a_{l,j}^{*} \hat{x}_j^{(p)}(k)\,\hat{x}_j^{(q)}(k-m) + \sum_{m=l}^{N_p-1} \sum_{q=1}^{N_u} \sum_{k=1}^{m+1} d_{j-1}^{(p)} a_{m,j-1} a_{l,j}^{*} \hat{x}_{j-1}^{(p)}(\beta-m-1+k)\,\hat{x}_j^{(p)}(k) + \\ & \sum_{m=0}^{l} \sum_{q=1}^{N_u} \sum_{k=\beta-m}^{\beta} d_{j+1}^{(p)} a_{m,j+1} a_{l,j}^{*} \hat{x}_{j+1}^{(q)}(k-\beta+m+1)\,\hat{x}_j^{(p)}(k) + \sum_{k=1}^{\beta} \xi_j(k)\,\hat{x}_j^{(p)}(k) \end{aligned} \tag{4.29}$$

$$rake(l)_j^{(p)} = D + MUI + Intra\ SI + Inter\ SI1 + Inter\ SI2 + \xi \tag{4.30}$$

The term *D* in equation (4.30) is the desired term containing information bit, *MUI* is the multi user interference caused by multi user, *Inter SI1* and *Inter SI2* are the inter symbol interferences caused by the previous information bit and next transmitted information bit respectively and *Intra SI* is the Intra symbol interference caused by the partial auto and cross correlation of the transmitted signal.

Decision Variable: $z_j^{(p)} = \sum_{l=0}^{N_p-1} rake(l)_j^{(p)}$

The values of the different parameters used for simulation are as given below:

Chip Duration, T_c = 0.8 µsec

Sampling Duration, T_s= 0.8 µsec

Sampling Frequency, $F_s = \frac{1}{T_s}$ =1.25 MHz

Path Delay, $\tau_n = n * T_c$

Doppler Shift, F_d = 20/40/80/160/320 Hz

Coherent Time, T_{ch} = 8.9/4.5/2.2/1.1/0.6 msec

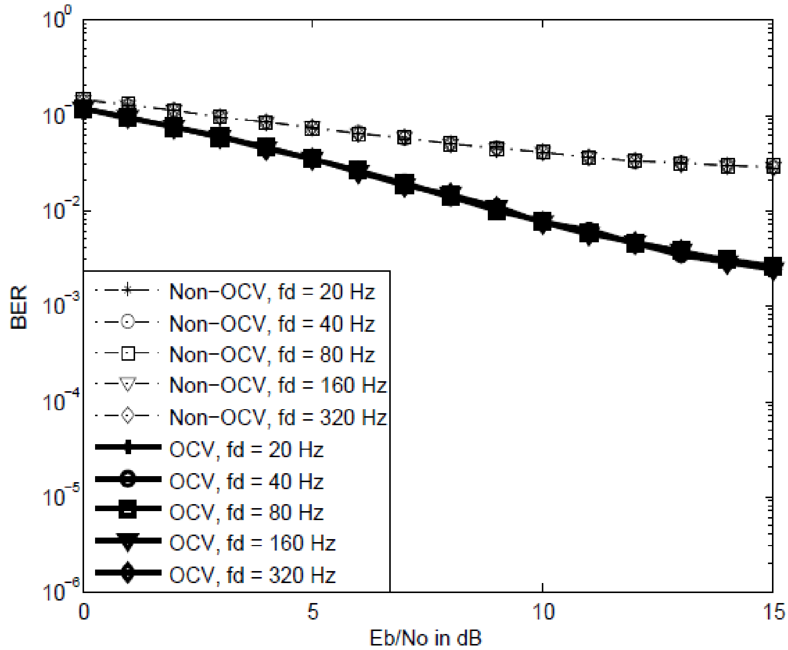
For simulation it is assumed that the delay of each path is equal to the integer multiple of chip rate and the channel coefficients are constant during the bit duration.

4.4.2. Simulation Results

In this section the simulated bit error rates of the multi user chaotic communication system using orthogonal chaotic vectors over fast frequency selective fading channel are presented and discussed. First the simulated bit error rates for the case when the channel coefficients are rayleigh distributed are presented then followed by the case when channel coefficients are Ricean distributed.

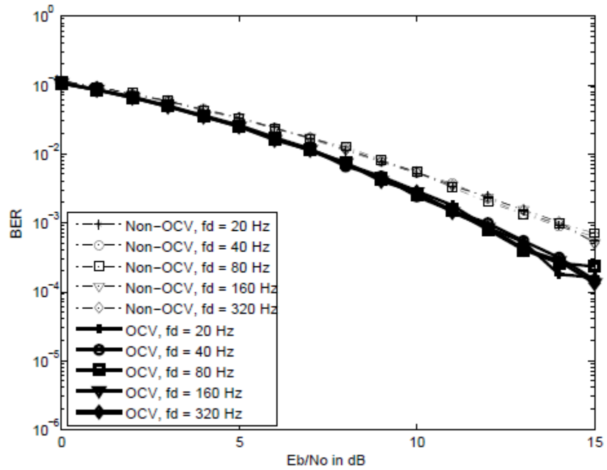
BER performance when channel coefficients are Rayleigh distributed:

Bit error rate performance of the Multiuser chaotic communication system over time varying frequency selective channel with channel coefficients being rayleigh distributed is simulated for different values of spread factor β and number of users, Nu. And the simulation results are shown in Fig. 4.24(a-d) for the case of number of multipath, N_p = 3. The simulated bit error rates are plotted for different doppler frequency when spread factor, β and number of users, N_u are fixed. From Fig. 4.24(a-d) it is observed that the doppler frequency has no effect on the bit error rate performance. Since, the coherent time, T_{ch} is greater than the symbol duration, T_{sym} = $\beta * T_c$.

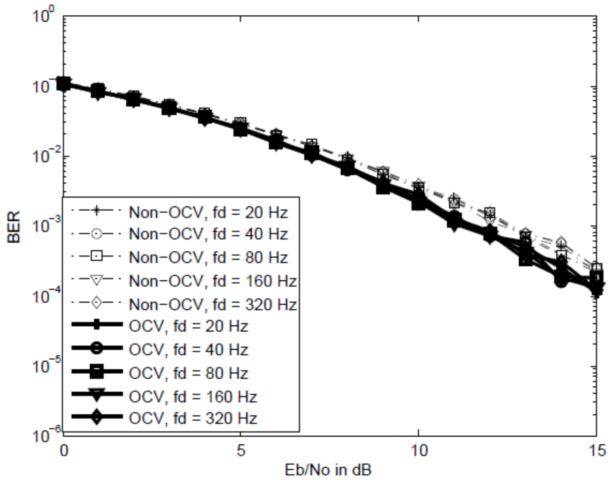


(a) Spread Factor, β = 16, Number Of Users, Nu = 4

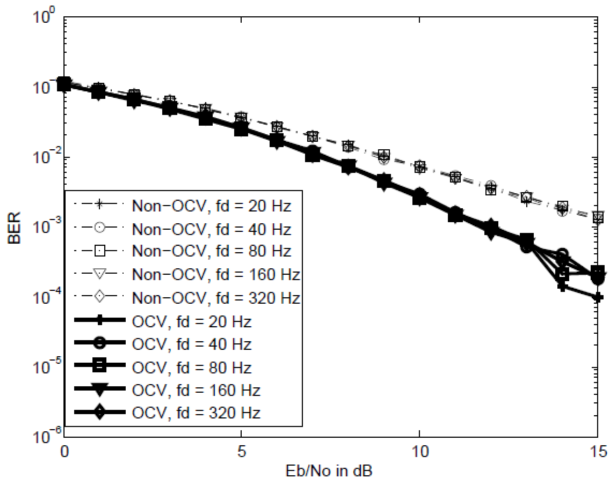
Fig. 4.24 (a-d). BER Performance of multi user chaotic system over rayleigh fast frequency selective fading channel for different values of spread factor, β and number of users, Nu



(b) Spread Factor, β =64, Number Of Users, Nu = 4



(c) Spread Factor, β = 256, Number Of Users, Nu = 8



(d) Spread Factor, β = 256, Number Of Users, Nu = 16

Fig. 4.24(a-d) BER Performance of multi user chaotic system over Rayleigh fast frequency selective fading channel for different values of spread factor, β and number of users, Nu (Continued)

Fig. 4.25 compares the bit error rate performance of the multi user chaotic communication system over Rayleigh fast frequency selective fading channel as a function of spread factor, β with constant E_b/N_o ratio. As spread factor, β is increased the bit error rate of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors decreases. The orthogonal chaotic vectors give better bit error rates than non-orthogonal chaotic vectors. In Fig. 4.26 the simulated bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors are plotted as a function of number of users, N_u.

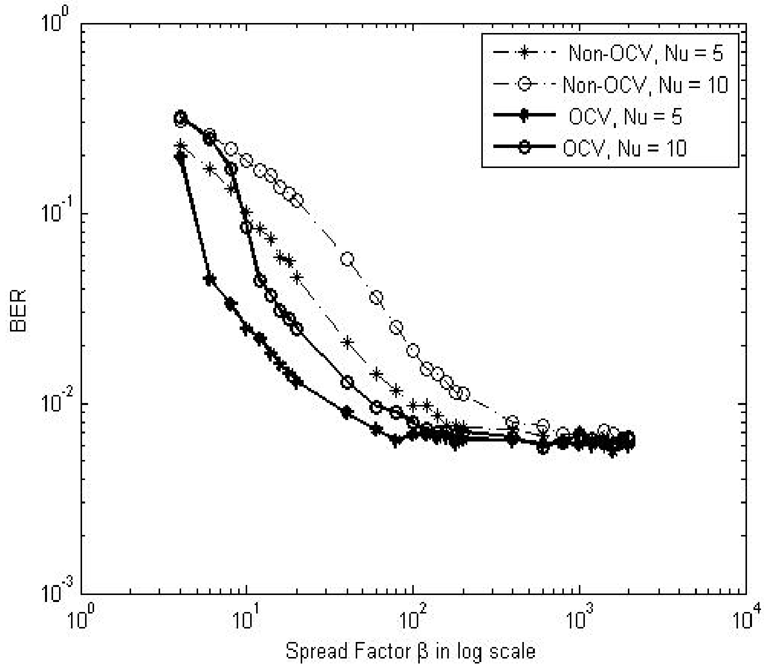


Fig. 4.25 BER v/s Spread Factor β for Number of Users Nu= 5 and 10 and Eb/No ratio 8 dB

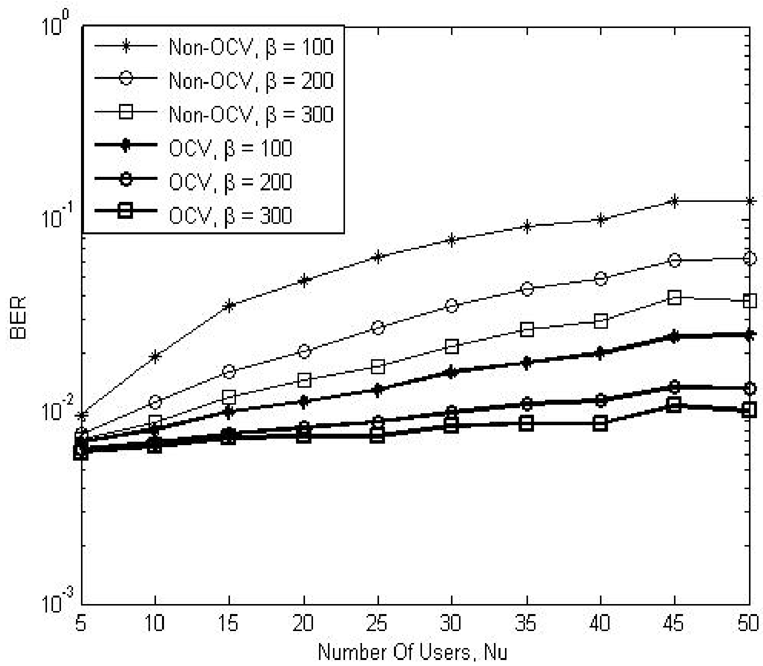
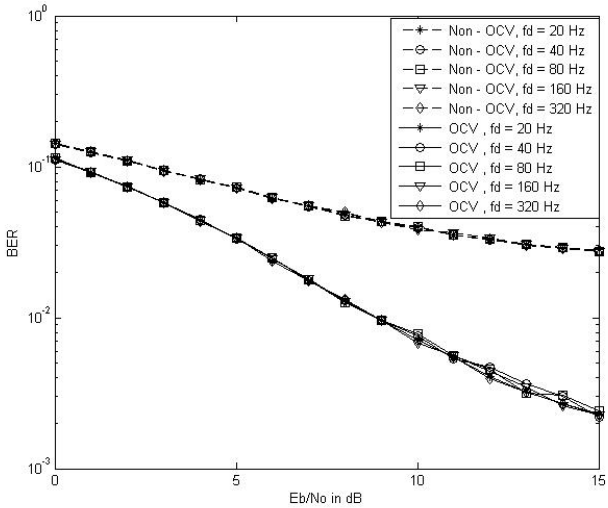


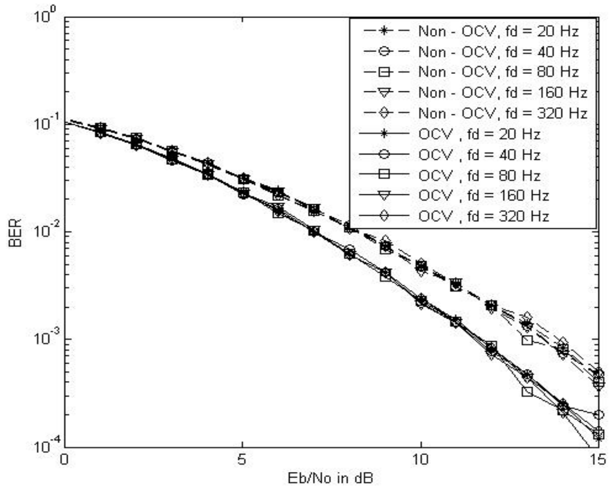
Fig. 4.26 BER v/s Number Of Users, Nu plots for Eb/No ratio 8 dB

BER performance when channel coefficients are Ricean distributed:

Bit error rate performance of the Multiuser chaotic communication system over time varying frequency selective channel with channel coefficients being Ricean distributed is simulated for different values of spread factor β and number of users, Nu. The simulation results are shown in Fig. 4.27(a-d) for the case of number of multipath, N_p = 3. The simulated bit error rates are plotted for different doppler frequency when spread factor, β and number of users, N_u are fixed. From Fig. 4.27(a-d) it is observed that the doppler frequency has no effect on the bit error rate performance. Since, the coherent time, T_{ch} is greater than the symbol duration, T_{sym} = $\beta * T_c$.

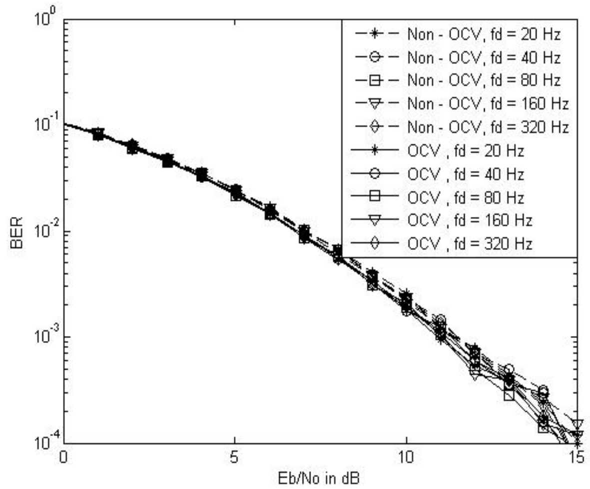


(a) Spread Factor,β = 16, Number Of Users, N_u = 4

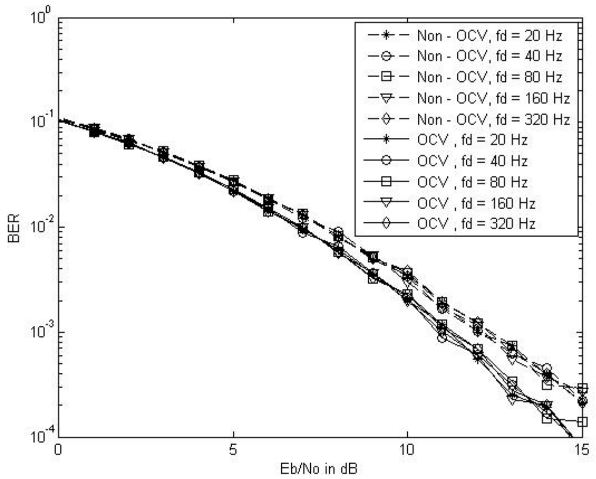


(b) Spread Factor,β = 64, Number Of Users, N_u = 4

Fig. 4.27 (a-d). BER Performance of multi user chaotic system over Ricean fast frequency selective fading channel for different values of spread factor, β and number of users, Nu



(c) Spread Factor,β = 256, Number Of Users, N_u = 4



(d) Spread Factor,β = 256, Number Of Users, N_u = 8

Fig. 4.27(a-d). BER Performance of multi user chaotic system over Ricean fast frequency selective fading channel for different values of spread factor, β and number of users, Nu(Continued)

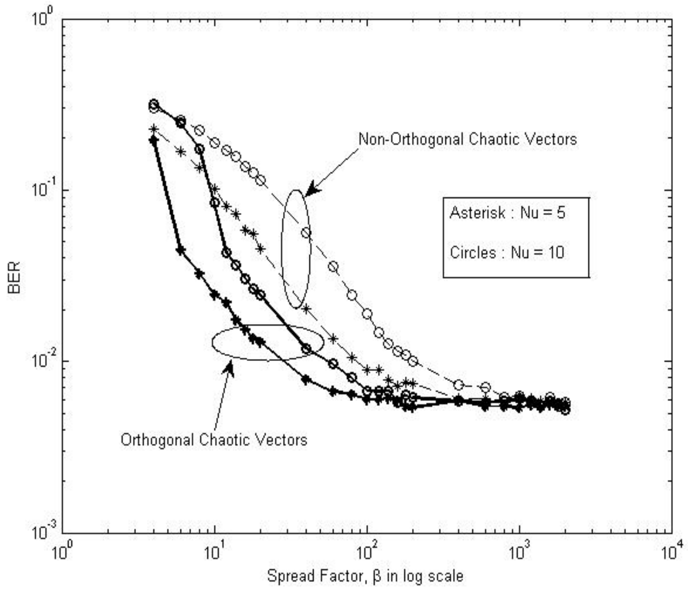


Fig. 4.28 BER v/s Spread Factor β for Number of Users Nu= 5 and 10 and Eb/No ratio 8 dB

Fig. 4.28 compares the bit error rate performance of the multi user chaotic communication system over Ricean fast frequency selective fading channel as a function of spread factor, β with constant E_b/N_o ratio. As spread factor, β is increased the bit error rate of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors decreases. The orthogonal chaotic vectors gives better bit error rates than non-orthogonal chaotic vectors. In Fig. 4.29 the simulated bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors and non-orthogonal chaotic vectors are plotted as a function of number of users, N_u.

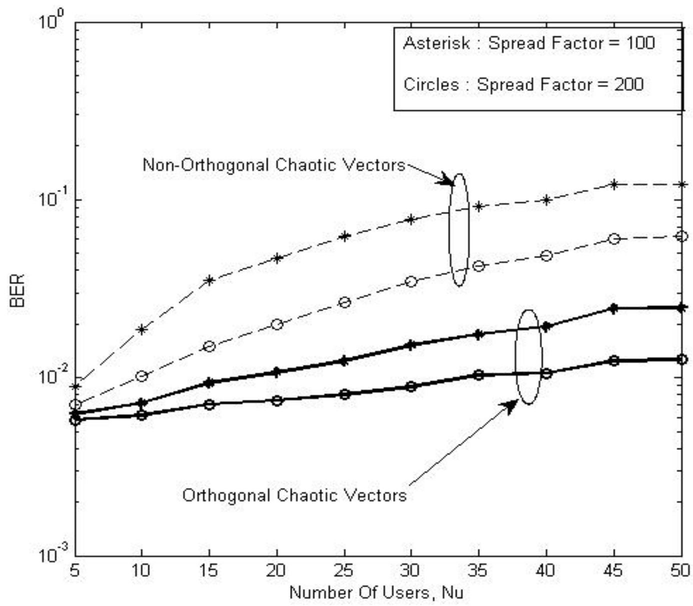


Fig. 4.29 BER v/s Number Of Users, Nu plots for Eb/No ratio 8 dB

4.5 Summary

- ✓ In this chapter the bit error rate performance of the multi user chaotic communication system using orthogonal chaotic vectors over multi path fading channel are analyzed through simulation.
- ✓ Theoretical equations for bit error rate are derived for flat fading condition. The bit error rate calculated using derived equations match convincingly with simulated bit error rates for larger values of spread factor, β and bit energy to noise ratio (E_b/N_o).
- ✓ Through simulations it is observed that the bit error performance of the multi user chaotic communication system using orthogonal chaotic vectors outperforms in multipath environment when compared to using non orthogonal chaotic vectors and conventional spreading sequences (Fig. 4.19 and Fig. 4.23).

5. Non Coherent Multi User Chaotic Communication System using Orthogonal Chaotic Vectors with Adaptive Multi User Receivers

Due to the lack of practical implementation of coherent communication schemes, non-coherent systems find advantage over coherent systems. A non-coherent multiple access communication system based on differential chaos shift keying (DCSK) was first proposed by Lau *et. al.* [33]. Later in [41], an improved non coherent multiple access scheme for chaos based communication system is discussed using two types of adaptive receivers: Adaptive Traversal Filter (ATF) based receiver and Inverse and Average (IA) receiver. Coulon et. al. [42], extended and generalized the work in [9], for both synchronous and asynchronous transmission and proposed four different adaptive receivers 1. Linear MMSE detectors 2. LMS detectors 3. Chaotic Sequence Estimator (CSE) 4. CSE – MMSE.

The quasi orthogonal characteristics of chaotic vector lead to residual cross correlation value which leads to multi user interference (MUI) in multi user environments. The effect of MUI due to quasi orthogonal characteristics of chaotic vectors can be reduced by using CSE-MMSE detector. In this chapter the bit error rate performance of the non-coherent multi user chaotic communication system using orthogonal chaotic vectors with adaptive multiple access receivers presented in [42] i.e., LMS detector and CSE based detector are evaluated using simulation and analytical expressions. The use of orthogonal chaotic vectors results in zero MUI and thus eliminates the additional processing of the received signal required for MUI reduction.

5.1 Transmitter Structure

Transmitter structure of the i^{th} user is shown in Fig. 5.1. The transmitter consists of an orthogonal chaotic vector generator, a group of delay blocks, switch and a multiplier. Data of each user is transmitted by modulating the orthogonal chaotic vector in the form of frames with the frame format for all users being identical. Each frame can be divided into two parts 1) T_b number of slots for training and 2) L number of slots for data.

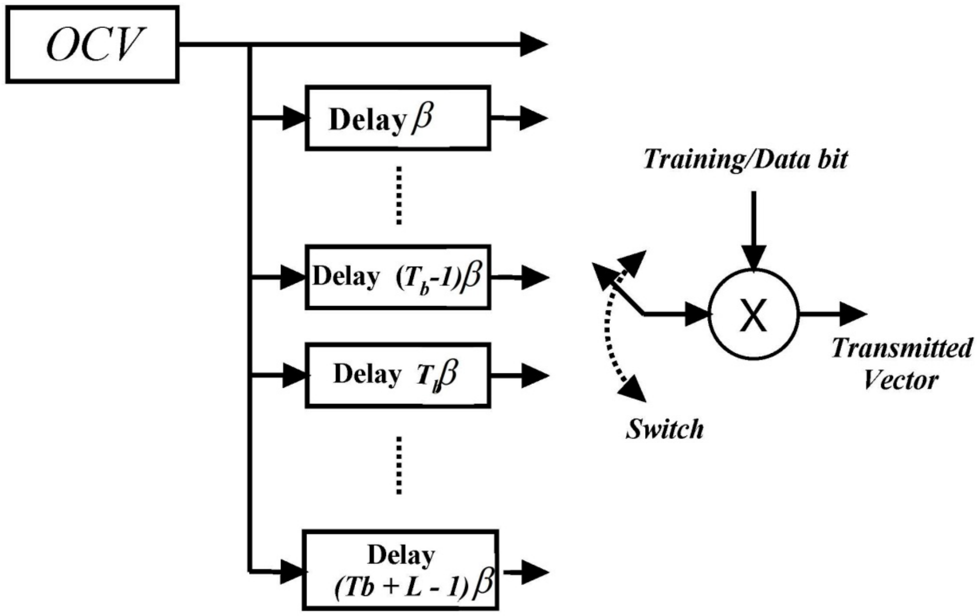


Fig. 5.1. Transmitter structure of the i^{th} user **[41]**

Assuming that the signal is corrupted only due to AWGN, l^{th} slot of the received signal $s_l(k)$ rcan be represented as

$$s_l(k) = \begin{cases} \sum_{i=1}^{N_u} T_{i,l} \hat{E}_b^{1/2} \hat{x}_i(k) + \xi_l(k) & l = 1 \ to \ T_b \\ \sum_{i=1}^{N_u} d_{i,l} \hat{E}_b^{1/2} \hat{x}_i(k) + \xi_l(k) & l = T_b + 1 \ to \ T_b + L \end{cases} \tag{5.1}$$

Where, $\hat{E}_b$ is the energy of the information bit at the correlator output and $\xi_l(k)$ represents the additive white Gaussian noise in the l^{th} slot with zero mean and variance $N_o/2$. $T_{i,l}$ and $d_{i,l}$ are the l^{th} training and data bits of i^{th} user respectively. If $\tilde{x}_j(k)$ is the estimated chaotic vector for j^{th} user. Then, the correlator output, $z_{j,n}$ for the $(n+1)^{th}$ data bit of j^{th} user is given by,

$$z_{j,n} = \sum_{k=1}^{\beta} \left\{ \left[\sum_{i=1}^{N_u} d_{i,n} \hat{E}_b^{1/2} \hat{x}_i(k) + \xi_n(k) \right] \left[\tilde{x}_j(k) \right] \right\} \tag{5.2}$$

Where, $n = 1,2,3, \ldots, L$

5.2 LMS/NLMS based adaptive multi user receiver

In LMS/NLMS based adaptive multi user receiver [41], [42], with the use of training sequence and LMS/NLMS algorithm the orthogonal chaotic vectors of each user are estimated iteratively.

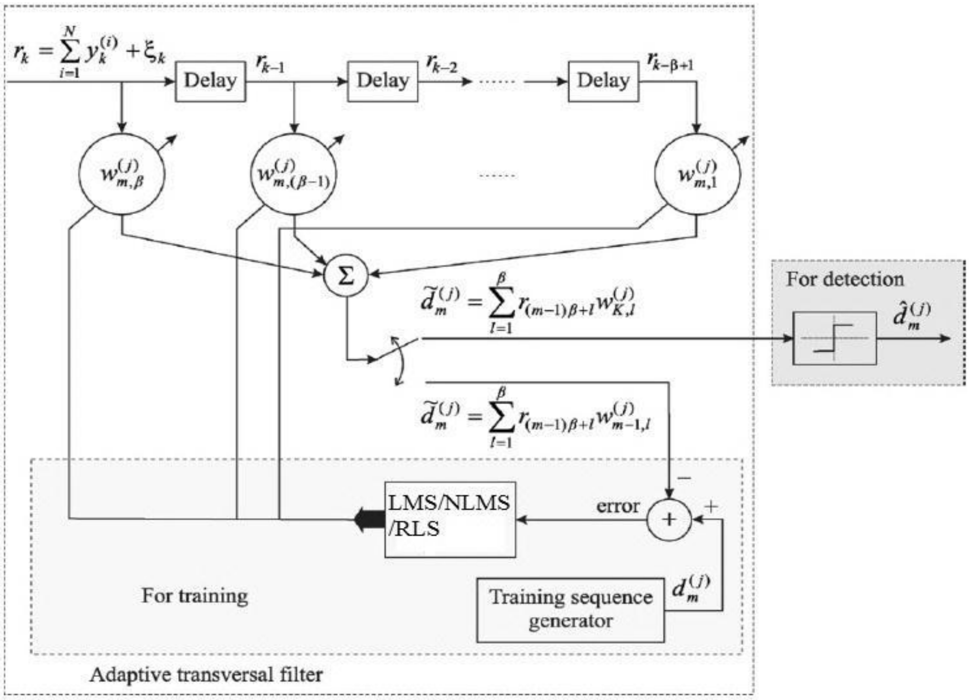


Fig. 5.2 Receiver Structure of non-coherent multi user chaotic communication system with adaptive receivers **[41]**

The LMS/NLMS update equations for j^{th} user are given by [52],

$$e_{j,l} = \left(\sum_{k=1}^{\beta} \tilde{x}_{j,l-1}(k) s_l(k) \right) - T_{j,l} \tag{5.3}$$

Where, m = 1,2, . . . ,T_b

$$\tilde{x}_{j,l}(k) = \tilde{x}_{j,l-1}(k) - \mu e_{j,l} s_l(k) \tag{5.4}$$

$$\tilde{x}_{j,l}(k) = \tilde{x}_{j,l-1}(k) - \mu' e_{j,l} s_l(k) \tag{5.5}$$

$$\mu' = \frac{\mu}{\gamma + \sum_{k=1}^{\beta} s_l(k)^2} . \tag{5.6}$$

Where equations (5.4) and (5.5) are update equations for LMS and NLMS algorithms respectively. μ is the step size and γ is the small offset value. For simulation the value of step size,μ and offset value, γ is set to 0.1 and 10^{-20} respectively. $\tilde{x}_{l,j}(k)$ is the estimated chaotic vector of j^{th} user after l iterations which is initially set to null vector.

5.2.1. *Bit Error Rate Analysis over AWGN Channel*

Assuming that for a sufficiently large number of training data the estimation error is negligible, then

$$\tilde{x}_j(k) \approx \hat{E}_b^{1/2} \hat{x}_j(k) \tag{5.7}$$

Ignoring the estimation error, the correlator output for j^{th} user for the n^{th} transmitted information bit is given by,

$$z_{j,n} = \sum_{k=1}^{\beta} \left\{ \left[\sum_{i=1}^{N_u} d_{i,n} \hat{E}_b^{1/2} \hat{x}_i(k) + \xi_n(k) \right] \left[\hat{E}_b^{1/2} \hat{x}_j(k) \right] \right\}$$
$$z_{j,n} = D + MUI + \xi \tag{5.8}$$

$$D = d_{j,n} \sum_{k=1}^{\beta} \left[\hat{E}_b^{1/2} \hat{x}_j(k) \right]^2$$

$$MUI = \sum_{i=1, i \neq j}^{N_u} d_{i,n} \sum_{k=1}^{\beta} \hat{E}_b \left(\hat{x}_i(k) \hat{x}_j(k) \right)$$

$$\xi = \sum_{k=1}^{\beta} \hat{E}_b^{1/2} \xi_n(k) \hat{x}_j(k)$$

In order to simplify the representation the subscript n can be ignored in further equations. Assuming, that $z^{(j)} \left| \left(d^{(j)} = -1 \right) \right.$ and $z^{(j)} \left| \left(d^{(j)} = +1 \right) \right.$ are random variables. According to central limit theorem for sum of large set of these random variables leads to Gaussian distribution. Therefore, BER for j^{th} user can be written as

$$BER^{(j)} = \frac{1}{2} erfc \left(\frac{E\left(z_{j,n} | d_{j,n} = +1 \right)}{\left(2 \operatorname{var}\left(z_{j,n} | d_{j,n} = +1 \right) \right)^{1/2}} \right) = \frac{1}{2} erfc \left(\frac{-E\left(z_{j,n} | d_{j,n} = -1 \right)}{\left(2 \operatorname{var}\left(z_{j,n} | d_{j,n} = -1 \right) \right)^{1/2}} \right) \tag{5.9}$$

Where, mean value of $\left(z_{j,n} | d_{j,n} = +1 \right)$ is given by,

$$E\left(z_{j,n} | d_{j,n} = +1 \right) = \beta \hat{E}_b E\left[\left(\hat{x}(k)_{j,n} \right)^2 \right] \tag{5.10}$$

And variance is given by,

$$\operatorname{var}\left(z_{j,n} | d_{j,n} = +1 \right) = \hat{E}_b^2 \operatorname{var}\left[\sum_{k=1}^{\beta} \left[\hat{x}_j(k) \right]^2 \right] + \hat{E}_b^2 \operatorname{var}\left[\sum_{i=1, i \neq j}^{N_u} \sum_{k=1}^{\beta} \left(\hat{x}_i(k) \hat{x}_j(k) \right) \right]$$
$$+ \beta \hat{E}_b E\left[\xi(k)^2 \right] E\left[\left(\hat{x}_j(k) \right)^2 \right] \tag{5.11}$$

Where,

$$E\left[\hat{x}_j(k) \right] = 0 \tag{5.12}$$

$$E\left[\left(\hat{x}_j(k) \right)^2 \right] = \frac{\psi}{\beta} = P_s \tag{5.13}$$

$$\operatorname{var}\left[\hat{x}_j(k) \right] = \frac{\psi}{\beta} \tag{5.14}$$

$$\operatorname{var}\left[\sum_{k=1}^{\beta} \left[\hat{x}_j(k) \right]^2 \right] = 0 \tag{5.15}$$

$$\operatorname{var}\left[\sum_{i=1,i\neq j}^{N_u}\sum_{k=1}^{\beta}\left(\hat{x}_i(k)\hat{x}_j(k)\right)\right]=\begin{cases}0 & for\ \ \beta > N_u\\ \beta\left(N_u-1\right)\mathrm{E}\left[\left(\hat{x}_i(k)\right)^2\right]\mathrm{E}\left[\left(\hat{x}_j(k)\right)^2\right] & for\ \ \beta \le N_u\end{cases} \tag{5.16}$$

$$\operatorname{var}\left[\xi(k)\right]=E\left[\xi(k)^2\right]=\frac{N_o}{2} \tag{5.17}$$

Where, P_s is average power transmitted by each user per bit, E_b is energy per bit and ψ is given by, $\psi=\beta\operatorname{var}[\hat{x}_j(k)]$. Since, the chaotic vectors are ortho normalized the value of ψ will be equal to 1.

Using the equations from (5.10) to (5.17) in (5.9), we get

$$BER^{(j)}=\begin{cases}\frac{1}{2}erfc\left[\left(\frac{2\left(N_u-1\right)}{\beta}+\left(\frac{\hat{E}_b}{N_o}\right)^{-1}\right)^{-\frac{1}{2}}\right] & for\ \ \beta\le N_u\\ \frac{1}{2}erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} & for\ \ \beta > N_u\end{cases} \tag{5.18}$$

$$BER^{(j)}_{LowerBound}=\frac{1}{2}erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} \tag{5.19}$$

Equation (5.18) gives the expression for bit error rate of j^{th} user. Since, the bits transmitted by all users are equi-probable the expression for average bit error rate for the multi user chaotic communication system using orthogonal chaotic vector will be,

$$BER=\begin{cases}\frac{1}{2}erfc\left[\left(\frac{2\left(N_u-1\right)}{\beta}+\left(\frac{\hat{E}_b}{N_o}\right)^{-1}\right)^{-\frac{1}{2}}\right] & for\ \ \beta\le N_u\\ \frac{1}{2}erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} & for\ \ \beta > N_u\end{cases} \tag{5.20}$$

$$BER_{LowerBound}=\frac{1}{2}erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} \tag{5.21}$$

5.2.2. *Simulation Results*

First the performance of the non-coherent multi user chaotic communication system using LMS based adaptive multi user receiver, then using NLMS based multi user receiver are tested through simulations.

LMS based Multi user Receiver (eq. 5.2.2)

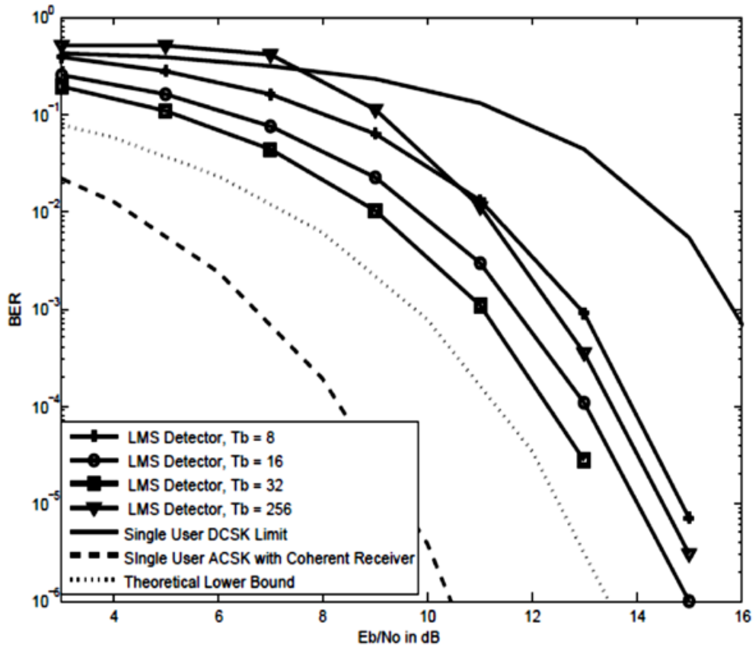


Fig. 5.3. BER v/s Eb/No with number of users Nu = 4 and spread factor 2β = 200

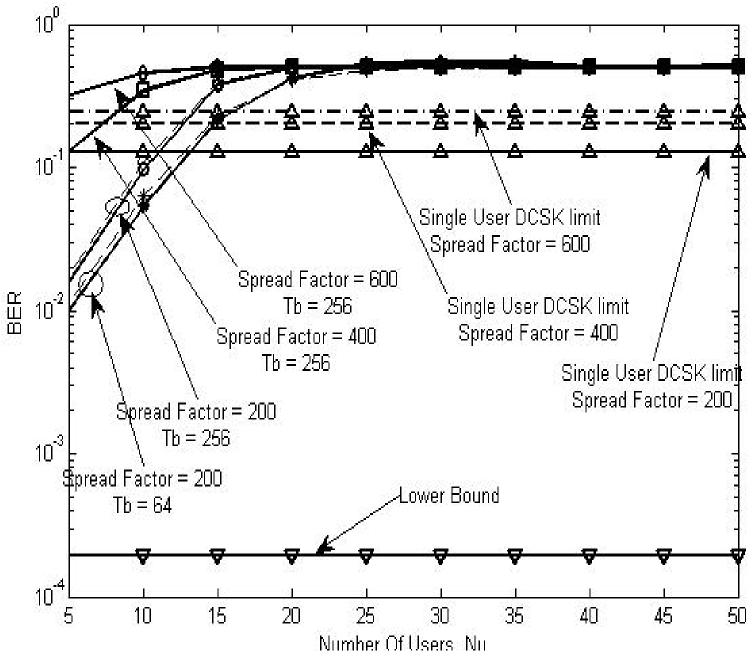


Fig. 5.4 BER v/s Number of users (Nu), for Eb/No = 11 dB, Solid lines : OCV, Dashed Lines : Non – OCV

In Fig. 5.3 the simulated bit error rates as function of bit energy to noise ratio (E_b/N_o) for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with LMS based adaptive multi user receiver for different number of training bits, T_b are plotted when number of users, N_u = 4 and spread factor, 2β = 200. For comparison the bit error rates of single user differential chaos shift keying (DCSK) communication system and single user antipodal chaos shift keying (ACSK) are also plotted in addition to the theoretical lower bound of bit error rate given in equation

(5.21). In Fig. 5.4 the simulated bit error rates for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with LMS based adaptive multi user receiver are plotted as a function of number of users for different values of spread factors, 2β and number of training bits, T_b. In this case it can be observed from Fig. 5.4 that the performance of using orthogonal chaotic vector or non-orthogonal chaotic vectors does not have any difference on bit error rate performance.

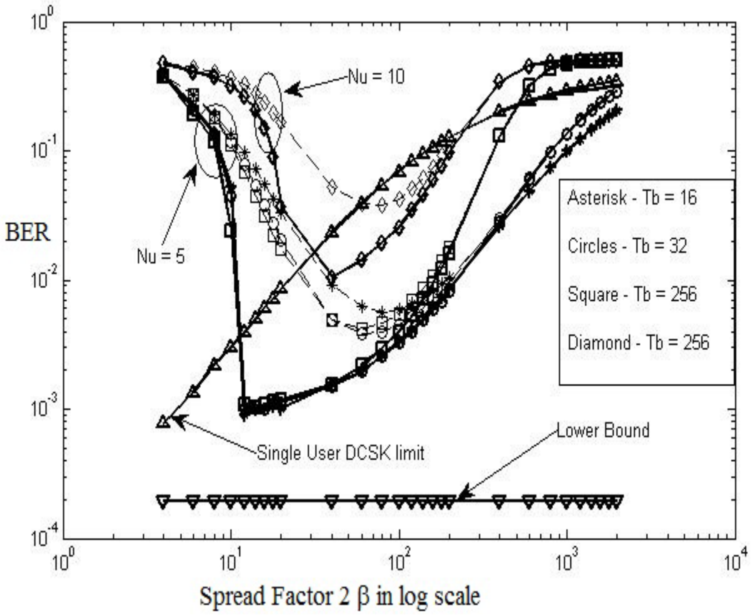


Fig. 5.5 BER v/s Spread factor (2β), with number of users Nu = 5 and Eb/No = 11 dB, Solid lines : OCV, Dashed Lines : Non – OCV

Fig. 5.5 compares the simulated bit error rate performance as a function of spread factor, 2β for different values of number of training bits, T_b used for training when LMS based adaptive multi user receivers are used. For smaller values of spread factor, 2β the use of orthogonal chaotic vectors shows a better bit error rate performance of non-coherent multi user chaotic communication system with LMS based adptive multi user receiver as compared to when non-orthogonal chaotic vectors are used. In Fig. 5.5 the theoretical lower bound for bit error rate and single user DCSK limit are also plotted for comparison.

Fig. 5.6 compares the simulated bit error rates when orthogonal chaotic vectors and non-orthogonalc chaotic vectors are used for non coherent multi user chaotic communication system with LMS based adaptive multi user receiver as a function of number of training bits, T_b used to train the adaptive multi user receiver. As number of users, N_u and spread factor, 2β are increased the bit error rate performance is degraded and divates away from the theoretical lower bound.

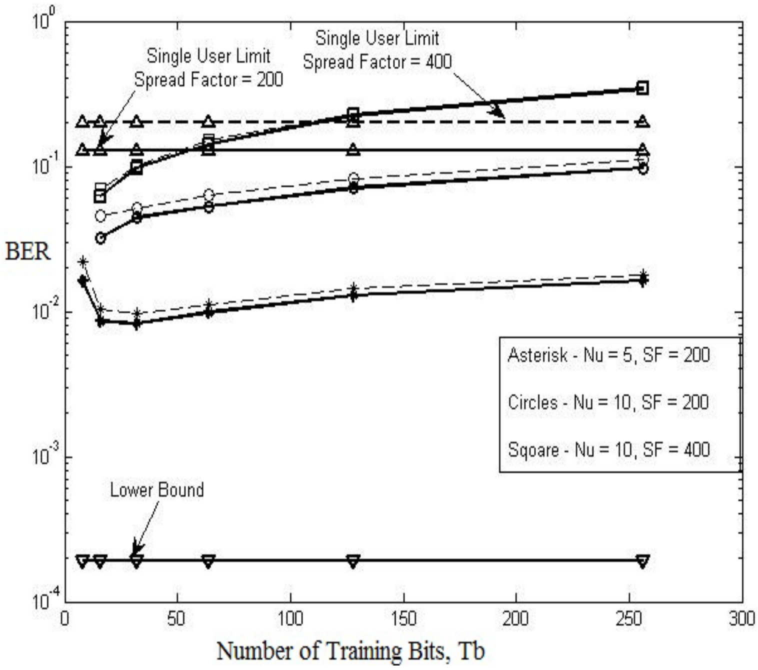


Fig. 5.6 BER v/s Number of training bits for Eb/No = 11 dB, Solid lines : OCV, Dashed Lines : Non – OCV

Normalized LMS based Multi user Receiver (5.2.3)

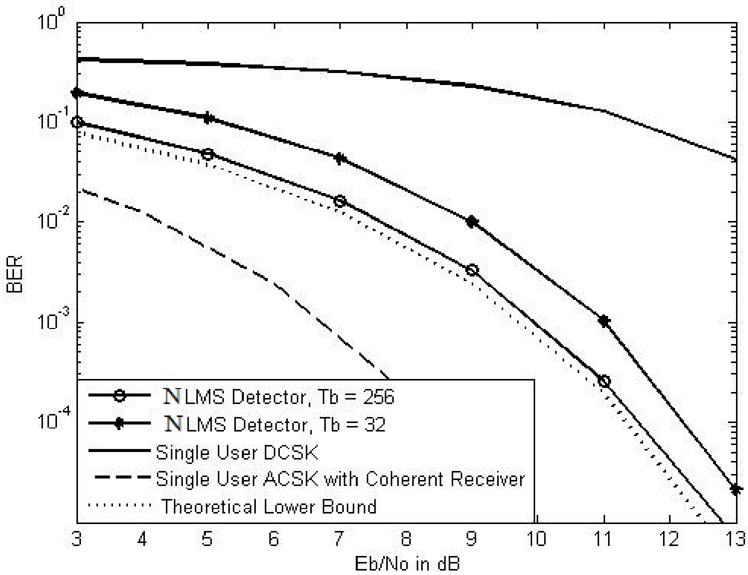


Fig. 5.7. BER v/s Eb/No with number of users Nu = 4 and spread factor 2β = 200

In Fig. 5.7 the simulated bit error rates as function of bit energy to noise ratio (E_b/N_o) for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with NLMS based adaptive multi user receiver for different number of training bits are plotted. For comparison the bit error rates of single user differential chaos shift keying (DCSK) communication system and single user antipodal chaos shift keying (ACSK) are also plotted in addition to the theoretical lower bound of bit error rate given in equation (5.2.1.15).As the number of training bits used to train the adaptive receiver in increased the bit error rate performance of the non-coherent multi user chaotic communication system using orthogonal chaotic vectors with NLMS based adaptive multi user receiver converges to the theoretical lower bound of bit error rate given by equation (5.21).

In Fig. 5.8 the simulated bit error rates for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with NLMS based adaptive multi user receiver are plotted as a function of number of users for different values of spread factors, 2β and number of training bits, T_b. From the Fig. 5.8 it is observed that the bit error performance is almost constant with increase in number of users, N_u when orthogonal chaotic vectors are used and the numbers of training bits used are large.

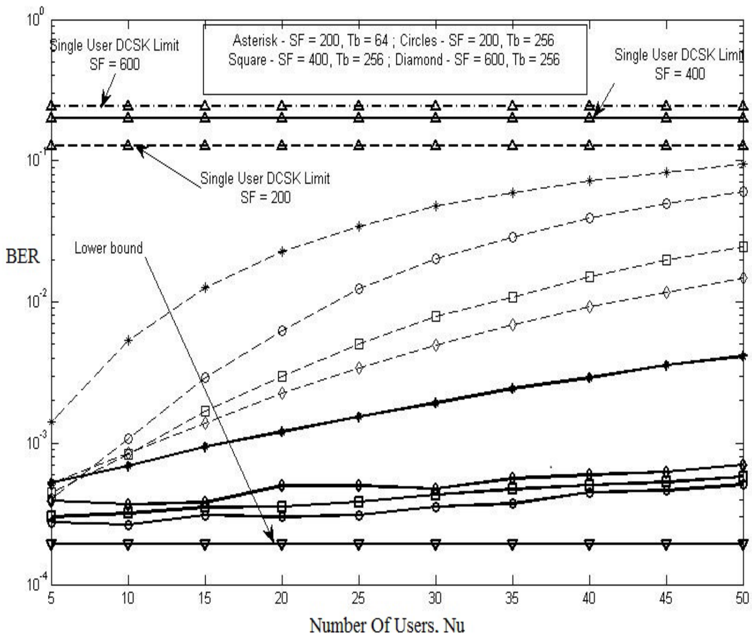


Fig. 5.8 BER v/s Number of users (Nu) for Eb/No = 11 dB, Solid Line: OCV, Dashed Line: Non-OCV

Fig. 5.9 compares the simulated bit error rate performance as a function of spread factor, 2β for different values of number of training bits,T_b used for training when NLMS based adaptive multi user receivers are used. From Fig. 5.9 it is observed that the theoretical lower bound for bit error rate can be attained using orthogonal chaotic vectors for spreading when sufficiently large number of training bits are used to train the adpative receiver provided the condition $\beta > N_u$ is satisfied. But as spread factor, 2β is increased the bit error rate deviates away from the theoretical lower bound and converges towards single user DCSK limit.

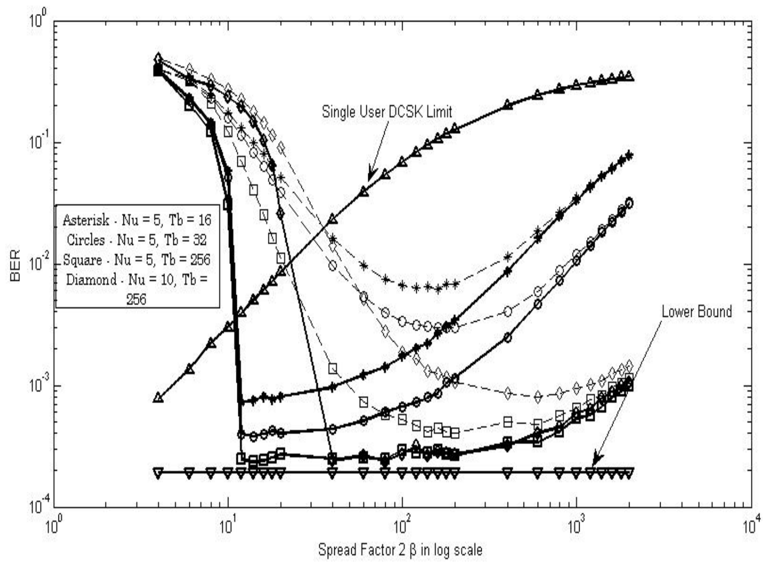


Fig. 5.9 BER v/s Spread factor (2β), for Eb/No = 11 dB, Solid Line : OCV, Dashed Line : Non – OCV

Fig. 5.10 compares the simulated bit error rates orthogonal chaotic vectors and non-orthogonalc chaotic vectors are used for non coherent multi user chaotic communication system with NLMS based adpaptive multis user receiver as a function of number of training bits, T_b used to train the adaptive multi user receiver. As we see in Fig. 5.10 the bit error rate performance tend to converge to the theoretical lower bound as the number of training bits, T_b is increased.

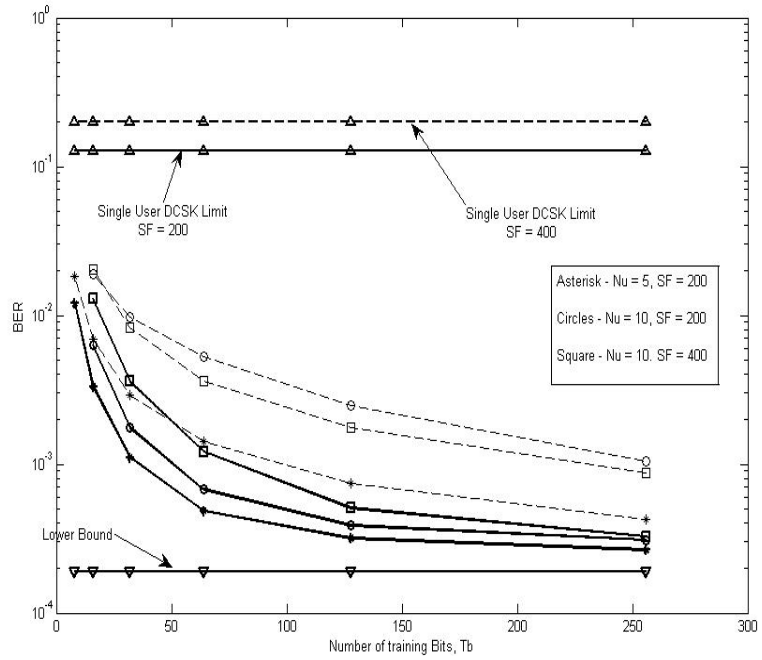


Fig. 5.10 BER v/s Number of training bits for Eb/No = 11 dB, Solid Line : OCV, Dashed Line : Non – OCV

5.3 RLS based Multi user Receiver

In this type of detector with the use of training sequence and RLS algorithm [52] the orthogonal chaotic vectors of each user are estimated iteratively. The estimated orthogonal chaotic vectors at correlator to extract the information bit.

5.3.1. *Bit Error Rate Analysis over AWGN channel*

Assuming that for a sufficiently large number of training data the estimation error is negligible, then

$$\tilde{x}_j(k) \approx \hat{E}_b^{1/2} \hat{x}_j(k) \tag{5.22}$$

Ignoring the estimation error, the correlator output for j^{th} user for the n^{th} transmitted information bit is given by,

$$z_{j,n} = \sum_{k=1}^{\beta} \left\{ \left[\sum_{i=1}^{N_u} d_{i,n} \hat{E}_b^{1/2} \hat{x}_i(k) + \xi_n(k) \right] \left[\hat{E}_b^{1/2} \hat{x}_j(k) \right] \right\}$$
$$z_{j,n} = D + MUI + \xi \tag{5.23}$$

$$D = d_{j,n} \sum_{k=1}^{\beta} \left[\hat{E}_b^{1/2} \hat{x}_j(k) \right]^2$$

$$MUI = \sum_{i=1, i \neq j}^{N_u} d_{i,n} \sum_{k=1}^{\beta} \hat{E}_b \left(\hat{x}_i(k) \hat{x}_j(k) \right)$$

$$\xi = \sum_{k=1}^{\beta} \hat{E}_b^{1/2} \xi_n(k) \hat{x}_j(k)$$

In order to simplify the representation the subscript *n* can be ignored in further equations. Assuming, that $z^{(j)} \middle| \left(d^{(j)} = -1 \right)$ and $z^{(j)} \middle| \left(d^{(j)} = +1 \right)$ are random variables. According to central limit theorem for sum of large set of these random variables leads to Gaussian distribution. Therefore, BER for j^{th} user can be written as

$$BER^{(j)} = \frac{1}{2} erfc \left(\frac{E\left(z_{j,n} \middle| d_{j,n} = +1 \right)}{\left(2\operatorname{var}\left(z_{j,n} \middle| d_{j,n} = +1 \right) \right)^{1/2}} \right) = \frac{1}{2} erfc \left(\frac{-E\left(z_{j,n} \middle| d_{j,n} = -1 \right)}{\left(2\operatorname{var}\left(z_{j,n} \middle| d_{j,n} = -1 \right) \right)^{1/2}} \right) \tag{5.24}$$

Where, mean value of $\left(z_{j,n} \middle| d_{j,n} = +1 \right)$ is given by,

$$E\left(z_{j,n} \middle| d_{j,n} = +1 \right) = \beta \hat{E}_b E\left[\left(\hat{x}(k)_{j,n} \right)^2 \right] \tag{5.25}$$

And variance is given by,

$$\operatorname{var}\left(z_{j,n} \middle| d_{j,n}=+1\right)= \hat{E}_b^2 \operatorname{var}\left[\sum_{k=1}^{\beta}\left[\hat{x}_j(k)\right]^2\right]+ \hat{E}_b^2 \operatorname{var}\left[\sum_{i=1,i\neq j}^{N_u}\sum_{k=1}^{\beta}\left(\hat{x}_i(k)\hat{x}_j(k)\right)\right] + \beta\hat{E}_b E\left[\xi(k)^2\right]E\left[\left(\hat{x}_j(k)\right)^2\right] \quad (5.26)$$

Where,

$$E\left[\hat{x}_j(k)\right]=0 \quad (5.27)$$

$$E\left[\left(\hat{x}_j(k)\right)^2\right]=\frac{\psi}{\beta}=P_s \quad (5.28)$$

$$\operatorname{var}\left[\hat{x}_j(k)\right]=\frac{\psi}{\beta} \quad (5.29)$$

$$\operatorname{var}\left[\sum_{k=1}^{\beta}\left[\hat{x}_j(k)\right]^2\right]=0 \quad (5.30)$$

$$\operatorname{var}\left[\sum_{i=1,i\neq j}^{N_u}\sum_{k=1}^{\beta}\left(\hat{x}_i(k)\hat{x}_j(k)\right)\right]=\begin{cases}0 & for\ \ \beta > N_u\\ \beta(N_u-1)\mathrm{E}\left[\left(\hat{x}_i(k)\right)^2\right]\mathrm{E}\left[\left(\hat{x}_j(k)\right)^2\right] & for\ \ \beta \le N_u\end{cases} \quad (5.31)$$

$$\operatorname{var}\left[\xi(k)\right]=E\left[\xi(k)^2\right]=\frac{N_o}{2} \quad (5.32)$$

Where, P_s is average power transmitted by each user per bit, E_b is energy per bit and ψ is given by, $\psi=\beta \operatorname{var}[\hat{x}_j(k)]$. Since, the chaotic vectors are ortho normalized the value of ψ will be equal to 1.

Using the equations from (5.25) to (5.32) in (5.24), we get

$$BER^{(j)}=\begin{cases}\frac{1}{2}erfc\left[\left(\frac{2(N_u-1)}{\beta}+\left(\frac{\hat{E}_b}{N_o}\right)^{-1}\right)^{-\frac{1}{2}}\right] & for\ \ \beta\le N_u\\ \frac{1}{2}erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} & for\ \ \beta > N_u\end{cases} \quad (5.33)$$

$$BER_{LowerBound}^{(j)}=\frac{1}{2}erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} \quad (5.34)$$

Equation (5.33) gives the expression for bit error rate of j^{th} user. Since, the bits transmitted by all users are equi-probable the expression for average bit error rate for the multi user chaotic communication system using orthogonal chaotic vector will be,

$$BER = \begin{cases} \frac{1}{2} erfc\left[\left(\frac{2(N_u - 1)}{\beta} + \left(\frac{\hat{E}_b}{N_o}\right)^{-1}\right)^{-\frac{1}{2}}\right] & for \;\; \beta \le N_u \\ \frac{1}{2} erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} & for \;\; \beta > N_u \end{cases} \tag{5.36}$$

$$BER_{LowerBound} = \frac{1}{2} erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} \tag{5.37}$$

5.3.2. *Simulation Results*

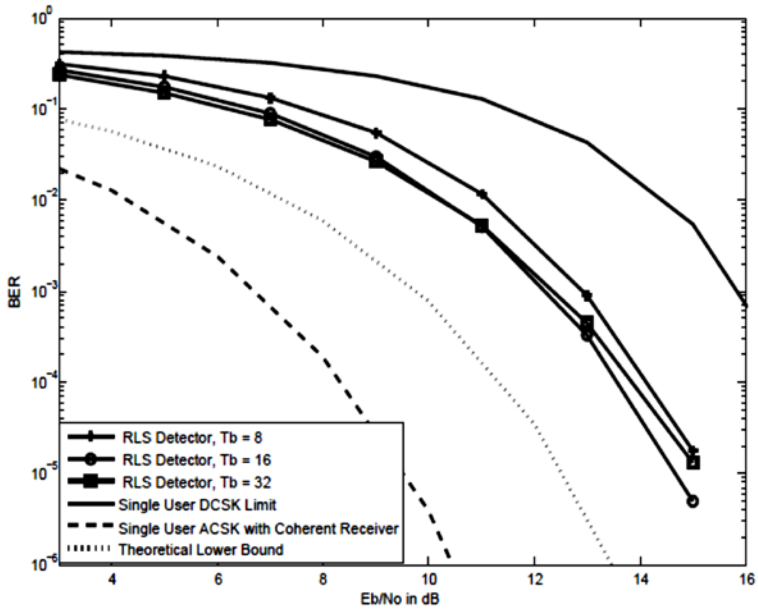


Fig. 5.11. BER v/s Eb/No with number of users Nu = 4 and spread factor 2β = 200

In Fig. 5.11 the simulated bit error rates as function of bit energy to noise ratio (E_b/N_o) for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with RLS based adaptive multi user receiver for different number of training bits, T_b are plotted. For comparison the bit error rates of single user differential chaos shift keying (DCSK) communication system and single user antipodal chaos shift keying (ACSK) are also plotted in addition to the theoretical lower bound of bit error rate given in equation (5.34).

In Fig. 5.12 the simulated bit error rates for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with RLS based adaptive multi user receiver are plotted as a function of number of users, N_u for different values of spread factors, 2β and number of training bits. In this case it can be observed that the performance when using orthogonal chaotic vector is almost constant whereas for the case of non-orthogonal vectors the performance deteriorates as the number of users, N_u increases.

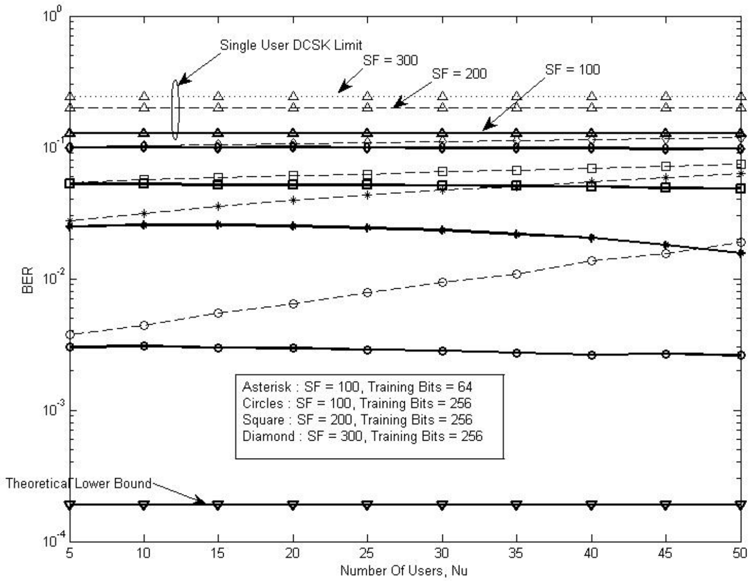


Fig. 5.12. BER v/s Number of users (Nu), with spread factor 2β = 200 and Eb/No = 11 dB

Fig. 5.13 compares the simulated bit error rate performance as a function of spread factor, 2β for different values of number of training bits,T_b used for training when RLS based adaptive multi user receivers are used. For smaller values of spread factor, 2β the use of orthogonal chaotic vectors shows a better bit error rate performance of non-coherent multi user chaotic communication system with RLS based adptive multi user receiver than compared to when non-orthogonal chaotic vectors are used. In Fig. 5.13 the theoretical lower bound for bit error rate and single user DCSK limit are also plotted for comparison.

Fig. 5.14 compares the simulated bit error rates orthogonal chaotic vectors and non-orthogonalc chaotic vectors are used for non-coherent multi user chaotic communication system with RLS based adpaptive multi user receiver as a function of number of training bits,T_b used to train the adaptive multi user receiver. As the number of training bits, T_b is increased the bit error rate initially decreases and starts increasing from T_b = 32 till T_b = 128 and then again decreases for the case of number of users, N_u = 5. When number of users, N_u = 10, the bit error rate initially decreases and starts increasing from T_b = 64.

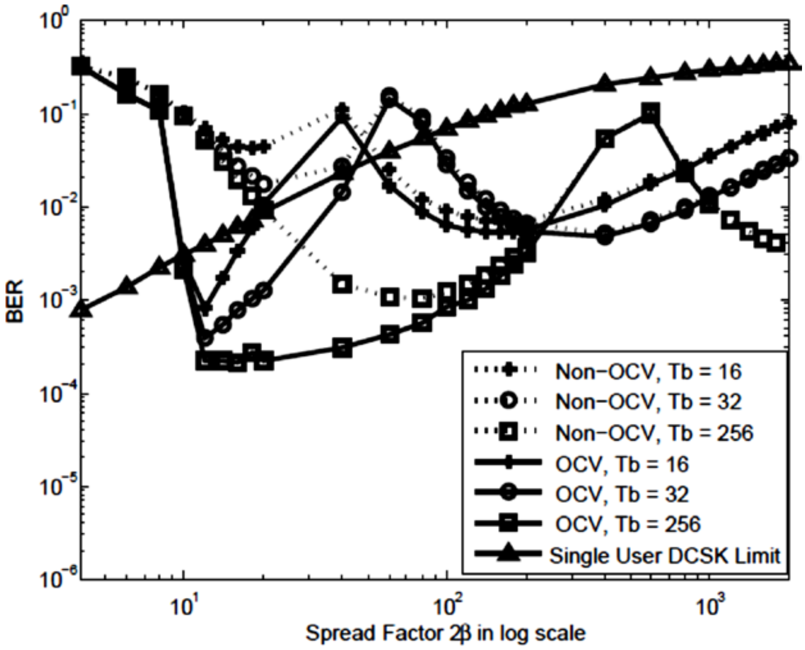


Fig. 5.13. BER v/s Spread factor (2β), with number of users Nu = 5 and Eb/No = 11 dB

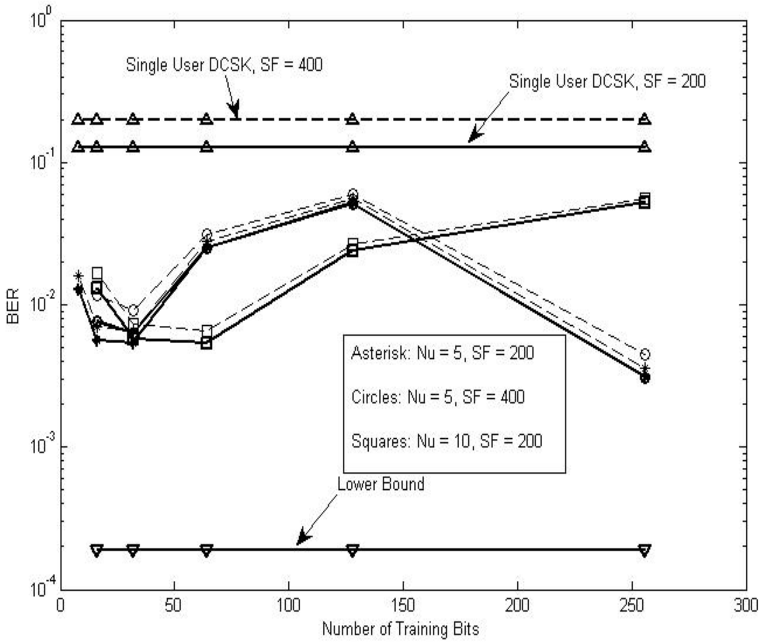


Fig. 5.14BER v/s Number of training bits for Eb/No = 11 dB, Solid Line : OCV, Dashed Line : Non – OCV

5.4 Inverse Averaging (IA) Receiver or Chaotic Sequence Estimator (CSE)

In IA [41] or CSE [42] based adaptive multi user receiver, the orthogonal chaotic vector is estimated by multiplying the training bit for corresponding user and then adding chaotic vectors from all the training slots and finally by dividing it with number of training bits.

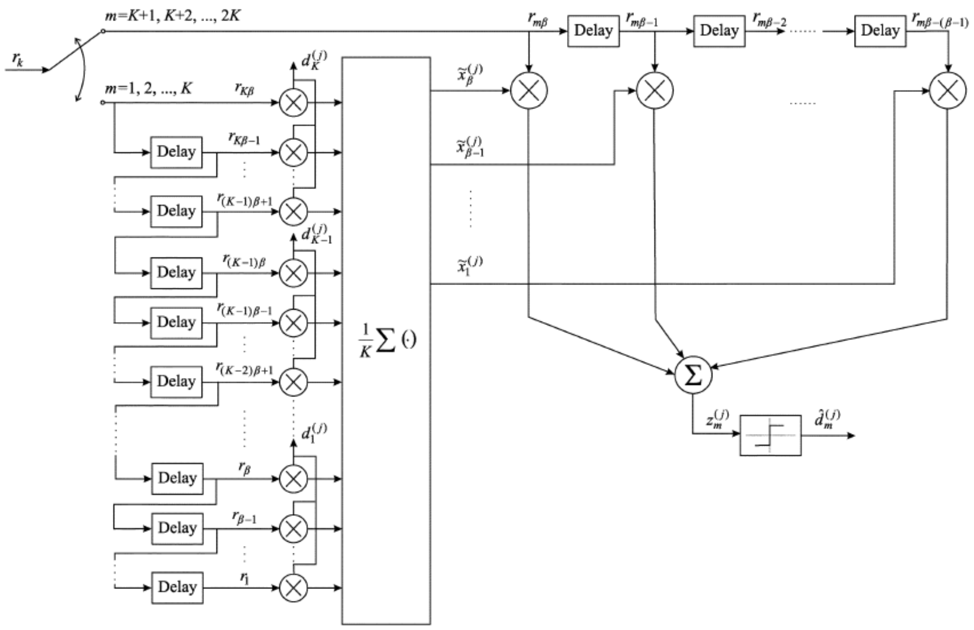


Fig. 5.15Receiver Structure of non-coherent multi user chaotic communication system with Inverse Averaging **[41]** based adaptive receivers

5.4.1. *Bit Error Rate Analysis over AWGN channel*

The estimated orthogonal chaotic vector $\tilde{x}_j(k)$ is given by,

$$\tilde{x}_j(k) = \frac{1}{T_b}\left\{\sum_{l=1}^{T_b} T_{j,l}\left[T_{j,l}\hat{E}_b^{1/2}\hat{x}_j(k) + \xi_l(k)\right]\right\} \tag{5.35}$$

$$\tilde{x}_j(k) = \hat{E}_b^{1/2}\hat{x}_j(k) + \frac{1}{T_b}\tilde{\xi}_j(k) \tag{5.36}$$

Where, $\tilde{\xi}_j(k) = \sum_{l=1}^{T_b} T_{j,l}\xi_l(k)$ is the noise component with zero mean and variance $T_b\left(\frac{N_o}{2}\right)$.

Then the correlator output is given by,

$$\begin{aligned} z_{j,n} = \sum_{k=1}^{\beta} d_{j,n}\left[\hat{E}_b^{1/2}\hat{x}_j(k)\right]^2 + \sum_{i=1, i\neq j}^{N_u}\sum_{k=1}^{\beta} d_{i,n}\hat{E}_b^{1/2}\hat{x}_i(k)\hat{E}_b^{1/2}\hat{x}_j(k) + \\ \sum_{k=1}^{\beta}\xi_n(k)\hat{E}_b^{1/2}\hat{x}_j(k) + \frac{1}{T_b}\sum_{i=1}^{N_u}\sum_{k=1}^{\beta} d_{i,n}\tilde{\xi}_j(k)\hat{E}_b^{1/2}\hat{x}_i(k) \\ + \frac{1}{T_b}\sum_{k=1}^{\beta}\tilde{\xi}_j(k)\xi_n(k) \end{aligned} \tag{5.37}$$

Assuming, that $z^{(j)}\left|\left(d^{(j)} = -1\right)\right.$ and $z^{(j)}\left|\left(d^{(j)} = +1\right)\right.$ are random variables. According to central limit theorem for sum of large set of these random variables leads to

Gaussian distribution. There BER for j^{th} user can be written as

$$BER^{(j)} = 0.5erfc\left(\frac{E\left[z_{j,n} \mid d_{j,n} = +1\right]}{\left(2\,\mathrm{var}\left[z_{j,n} \mid d_{j,n} = +1\right]\right)^{1/2}}\right) = 0.5erfc\left(\frac{-E\left[z_{j,n} \mid d_{j,n} = -1\right]}{\left(2\,\mathrm{var}\left[z_{j,n} \mid d_{j,n} = -1\right]\right)^{1/2}}\right) \quad (5.38)$$

$$E\left[z_{j,n} \mid d_{j,n} = +1\right] = \beta \hat{E}_b E\left[\hat{x}_j(k)^2\right] = \hat{E}_b \quad (5.39)$$

Where, $\hat{E}_b$ denotes the energy per bit in the demodulation process.

$$\begin{aligned}\mathrm{var}\left[z_{j,n} \mid d_{j,n} = +1\right]_{\beta<N_u} = {} & \frac{\beta(N_u-1)}{2}\hat{E}_b^2 E\left[\hat{x}_j(k)^2\right]E\left[\hat{x}_i(k)^2\right] + \beta\frac{N_o}{2}\hat{E}_b E\left[\hat{x}_j(k)^2\right] + \\ & \frac{N_u\beta N_o}{2T_b}\hat{E}_b E\left[\hat{x}_i(k)^2\right] + \frac{\beta}{T_b}\left(\frac{N_o}{2}\right)^2\end{aligned} \quad (5.40)$$

$$\begin{aligned}\mathrm{var}\left[z_{j,n} \mid d_{j,n} = +1\right]_{\beta\geq N_u} = {} & \beta\frac{N_o}{2}\hat{E}_b E\left[\hat{x}_j(k)^2\right] + \\ & \frac{N_u\beta N_o}{2T_b}\hat{E}_b E\left[\hat{x}_i(k)^2\right] + \frac{\beta}{T_b}\left(\frac{N_o}{2}\right)^2\end{aligned} \quad (5.4.1)$$

Using eq. (5.4.1.5), (5.4.1.6) and (5.4.1.7) in eq. (5.4.1.4)

$$BER^{(j)} = \begin{cases} 0.5erfc\left(\left[\left(\frac{\hat{E}_b}{N_o}\right)^{-1}\left[1 + \frac{N_u}{T_b} + \frac{\beta}{2T_b}\left(\frac{\hat{E}_b}{N_o}\right)^{-1}\right]\right]^{-\frac{1}{2}}\right) & for\ \beta > N_u \\ 0.5erfc\left(\left[\left(\frac{\hat{E}_b}{N_o}\right)^{-1}\left[1 + \frac{N_u}{T_b} + \frac{\beta}{2T_b}\left(\frac{\hat{E}_b}{N_o}\right)^{-1}\right] + \frac{2(N_u-1)}{\beta}\right]^{-\frac{1}{2}}\right) & for\ \beta \leq N_u \end{cases} \quad (5.42)$$

$$BER^{(j)}_{LowerBound} = 0.5erfc\left(\frac{\hat{E}_b}{N_o}\right)^{\frac{1}{2}} \quad (5.43)$$

5.4.2. *Simulation Results*

In Fig. 5.16 the simulated bit error rates as function of bit energy to noise ratio (E_b/N_o) for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with CSE based adaptive multi user receiver for different number of training bits, T_b are plotted. For comparison the bit error rates of single user differential chaos shift keying (DCSK) communication system and single user antipodal chaos shift keying (ACSK) are also plotted in addition to the theoretical lower bound of bit error rate given in equation (5.43).The simulated bit error rates are also compared with that of the theoretical bit error rate computed using equation (5.44). It is observed that the theoretical and the simulated bit error rates exactly match with each other.

In Fig. 5.17 the simulated bit error rates for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with CSE based adaptive multi user receiver are plotted as a function of number of users, N_u. The simulated bit error rates are also compared with theoretical bit error rate and theoretical lower bound computed using equations (5.42) and (5.43) respectively. It is observed that the theoretical and the simulated bit error rates are in exact match.

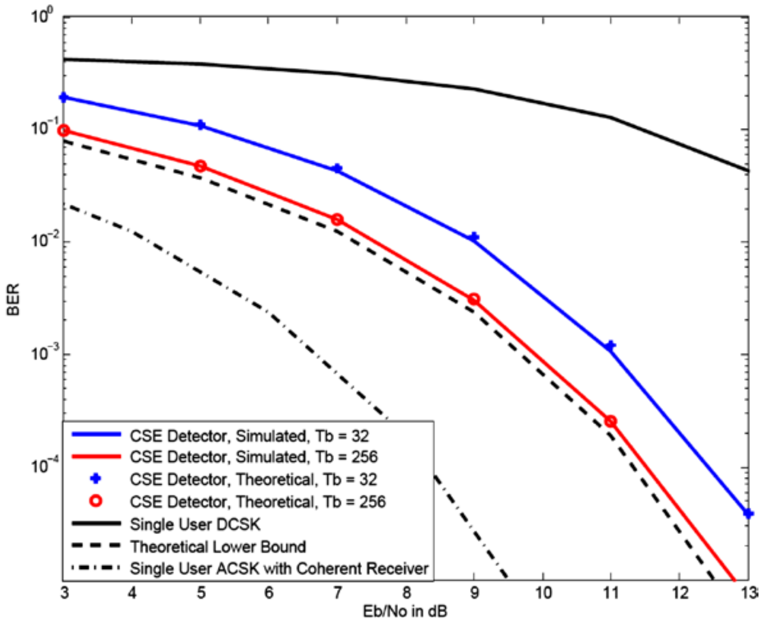


Fig. 5.16. BER v/s Eb/No with number of users Nu = 4 and spread factor 2β=200

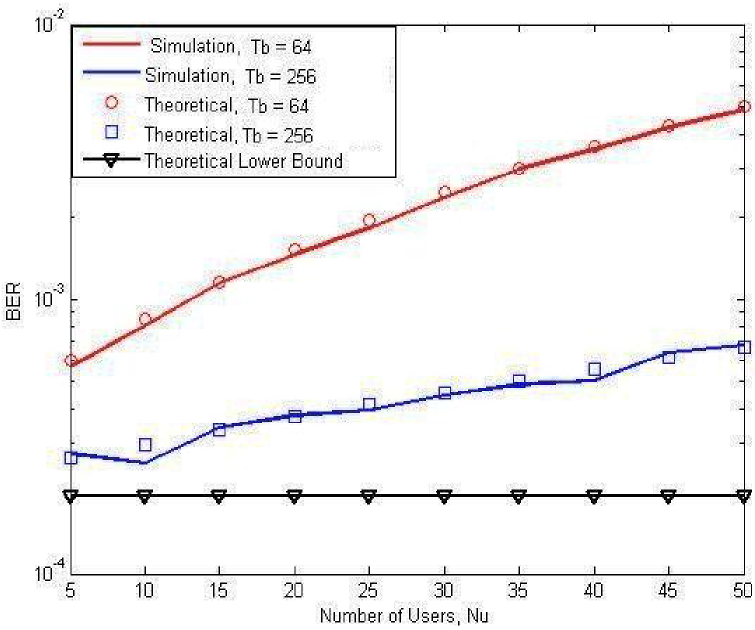


Fig. 5.17 BER v/s Number of users plot for Eb/No = 11 dB and spread factor, 2β = 200

Fig. 5.18 compares the simulated bit error rate performance as a function of spread factor, 2β for different values of number of training bits, T_b used for training when

CSE based adaptive multi user receivers are used. For smaller values of spread factor, 2β the use of orthogonal chaotic vectors shows a better bit error rate performance of non-coherent multi user chaotic communication system with CSE based adptive multi user receiver than compared to when non-orthogonal chaotic vectors are used. In Fig. 5.18 the theoretical bit error rate and theoretical lower bound are also plotted which are computed using equation (5.42) and (5.43).

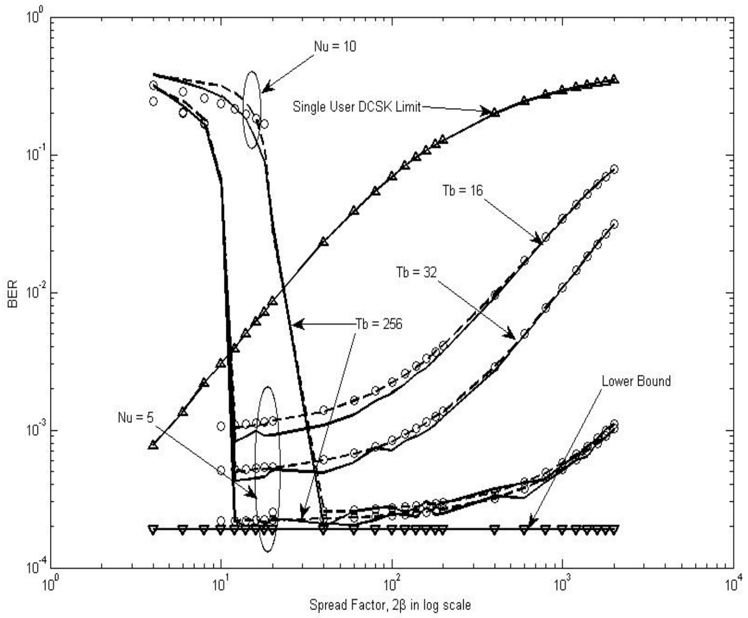


Fig. 5.18 BER v/s Spread Factor 2β for Eb/No = 11 dB, Solid Line: Simulation, Circles: Theoretical, Dashed Line: MAS

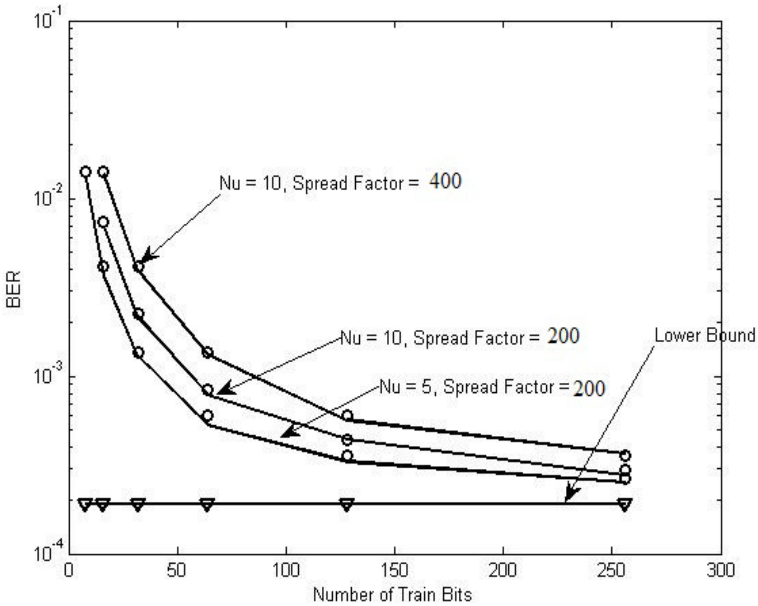


Fig. 5.19. BER v/s Training bits for Eb/No = 11 dB, Solid line: Simulation and Circles: Theoretical

Fig. 5.19 compares the simulated bit error rates when are used for non coherent multi user chaotic communication system with CSE based adpaptive multi user receiver as a function of number of training bits, T_b. As number of users, N_u and spread factor, 2β are increased the bit error rate performance is degraded but as the number of training bits, T_b is icreased the bit error rate converges towards theoretical lower bound.

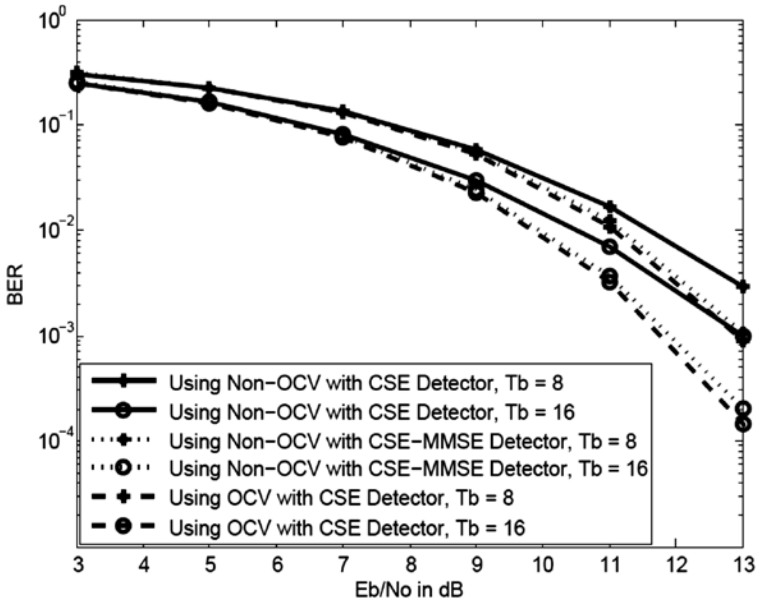


Fig. 5.20. BER v/s Eb/No with number of users Nu = 4 and spread factor 2β = 200

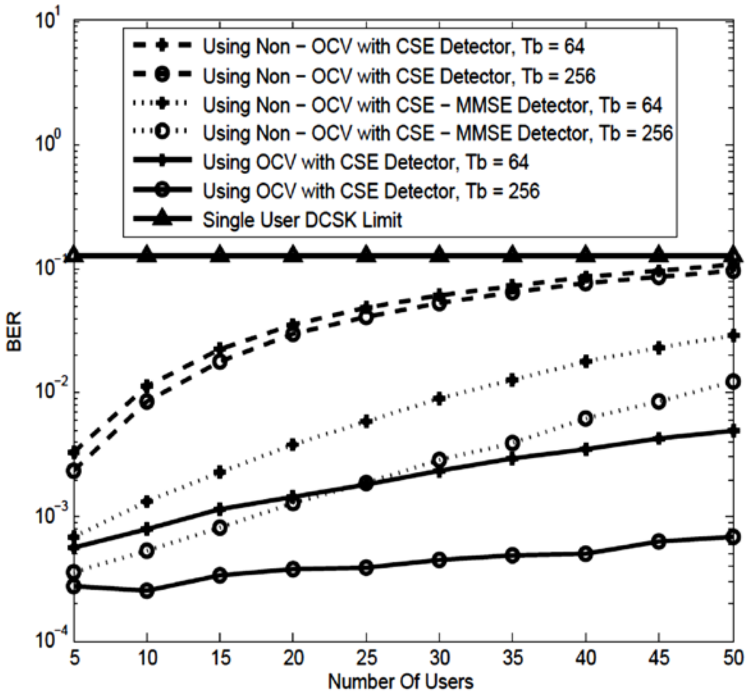


Fig. 5.21. BER v/s Number of users (Nu), with spread factor 2β = 200 and Eb/No = 11 dB

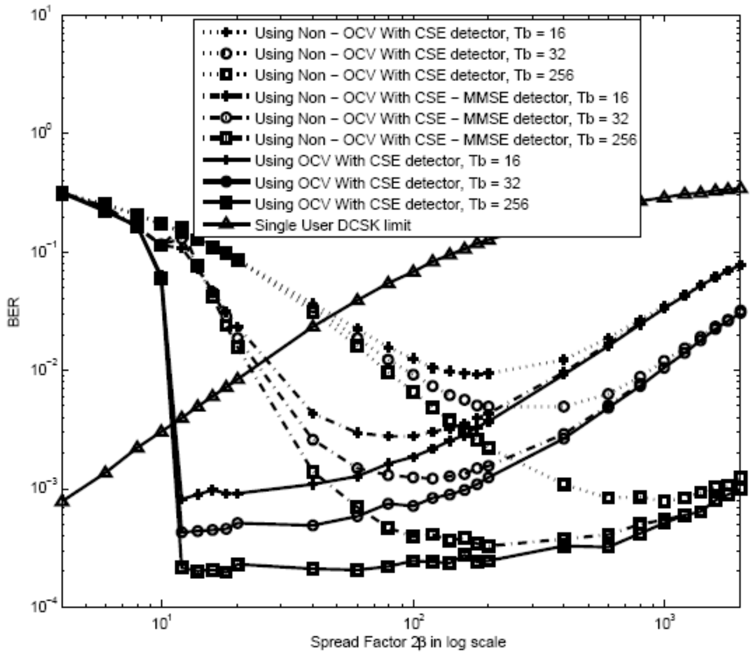


Fig. 5.22. BER v/s Spread factor (2β), with number of users Nu = 5 and Eb/No = 11 dB

From Fig. 5.20 to Fig. 5.22 the simulated bit error rates for the non-coherent multi user chaotic communication system using non-orthogonal chaotic vectors with CSE and CSE-MMSE based multi user receivers and using orthogonal chaotic vectors with CSE based multi user receiver. CSE-MMSE based multi user receivers are used to eliminate the MUI by performing MMSE equalization on estimated chaotic vector. From Fig. 5.20 to Fig. 5.22 it is observed that the bit error rate performance of the non-coherent multi user chaotic communication system using orthogonal chaotic vector with CSE based multi user receiver outperforms when compared to using non-orthogonal chaotic vectors with CSE and CSE-MMSE based multi user receiver.

From Fig. 5.23 to Fig. 5.26 the simulated bit error rates for non-coherent multi user chaotic communication system using orthogonal and non-orthogonal chaotic vectors with different kinds of multi user receivers discussed in this chapter (LMS/NLMS/RLS/CSE/CSE-MMSE based multi user receivers) are compared. From Fig. 5.23 to Fig. 5.26 it is observed that using orthogonal chaotic vectors for spreading the data leads to the better bit error rate performance than compared to its counter part using non-orthogonal chaotic vectors

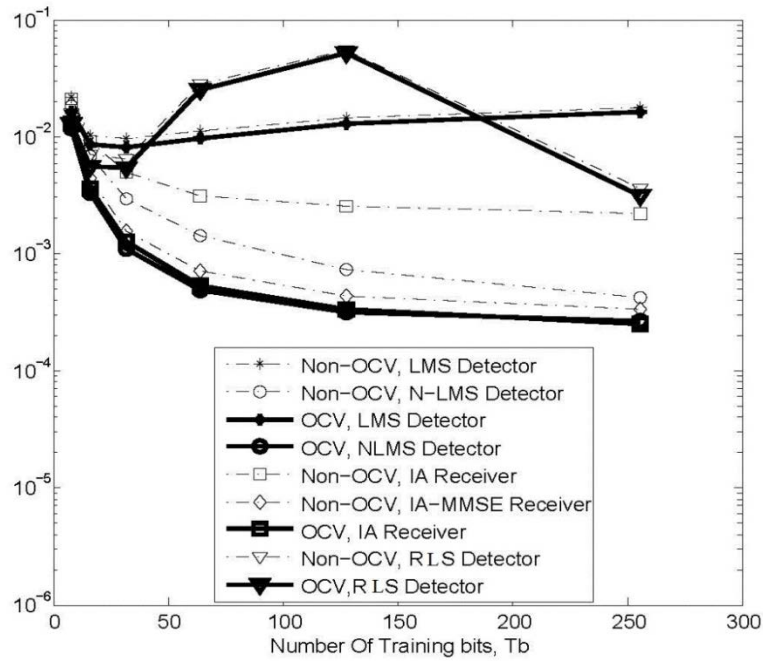


Fig. 5.23.Comparison of BER v/s Training bit length for Spread factor, 2β = 200, Number of users Nu = 5, Eb/No = 11dB

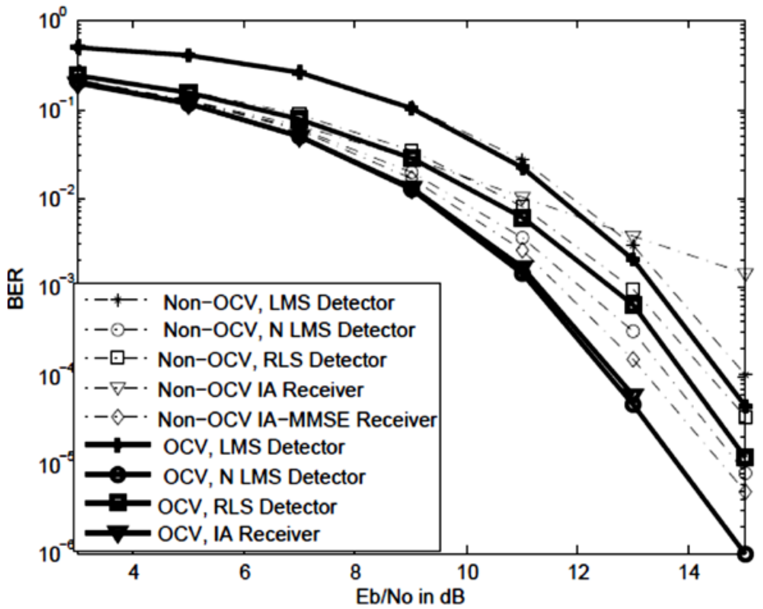


Fig. 5.24.Comparison of BER v/s Eb/No plots for spread factor, 2β = 200, Number of Users Nu = 5 and training bits, Tb = 32

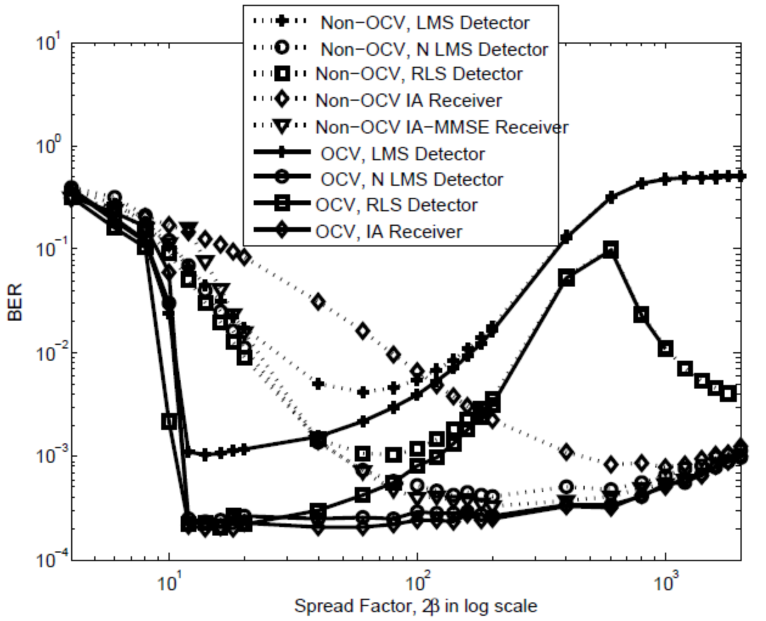


Fig. 5.25. Comparison of BER v/s Spread factor, 2β for Number of Users,Nu = 5 and training bits,Tb = 256, Eb/No = 11 dB

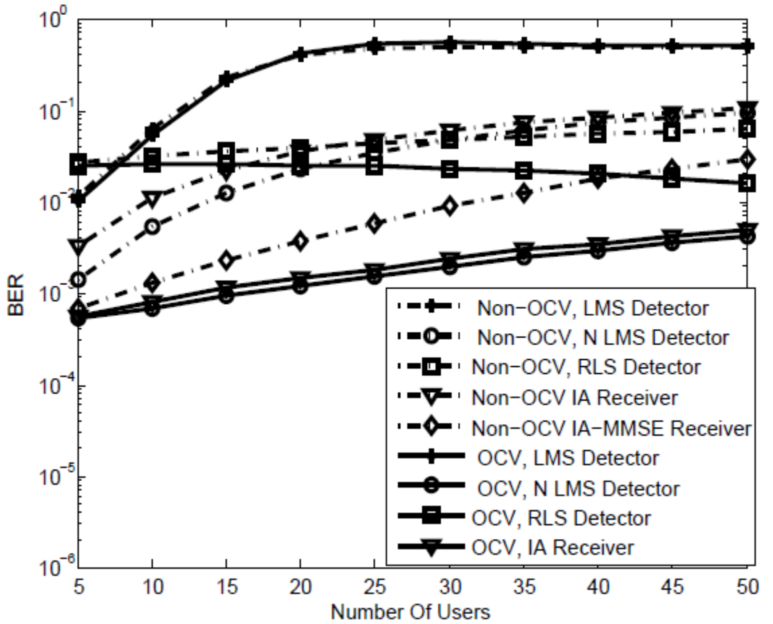


Fig. 5.26.Comparison Of BER v/s Number of users, Nu for Spread factor, 2β = 100,Training bits, Tb = 64, and Eb/No = 11 dB

5.5 Discussion

Training Sequence:

The convergence time of the adaptive sequence estimators depends on the choice of training sequence. Using orthogonal training sequence for each user will reduce the convergence time for LMS/NLMS/RLS based multi user receivers. The equation for bit error rate for non-coherent multi user chaotic communication system using orthogonal chaotic vectors with CSE/IA based multi user receiver is derived by considering that the training sequence for each user are orthogonal. If the non-orthogonal training sequences are used then the bit error rate performance converges to equation (5.4.1.9) when length of the training sequence is very large. The advantage of using orthogonal codes as training sequences is lost if the transmission is asynchronous.

All simulations work documented in this chapter are done by using Walsh-hadamard codes as training sequence.

Bit Energy to Noise ratio:

In equations (5.21), (5.37) and (5.43) the BER's are expressed in terms of $\hat{E}_b$, which is the energy per bit in demodulation process. If $E_b^{(j)}$ is the average bit energy of the j^{th} user. Then $E_b^{(j)}$ and $\hat{E}_b$ are related by, $E_b^{(j)} = 2\,\hat{E}_b$.

All BER plots in this paper are plotted with respect to average bit energy E_b to noise ratio E_b/N_o and it is assumed that the energy per bit of all users are equal. If $\hat{E}_b$ is used instead of E_b then the BER plots will be same but with -3 dB shift.

Spread Factor:

Each slot in the transmit frame consists of β number of chaotic samples which are modulated by either training bit or data bit. Assuming the length of training sequence T_b is equal to the length of data sequence in each frame L. Under this assumption it requires 2β samples to transmit one bit of information. Thus the average spreading factor per bit is equal to 2β.

Bit Error Rate

In the derivation of BER equation for LMS/NLMS/RLS based detector the estimation error is ignored. If sufficiently long training sequence is used then the estimation error will be negligible. Therefore, the equations derived for the case of LMS/NLMS and RLS based multi user receivers is just a theoretical lower bound which can be attained only when the estimation error is zero and using orthogonal chaotic vectors as spreading sequences.

Synchronous and asynchronous transmission

The transmission is said to be as synchronous if the transmission of data from all users is done at same instant else the transmission is said to be as asynchronous.

Orthogonal Chaotic Vectors(OCV) can't be used when the transmission is asynchronous. Since, under the asynchronous transmission orthogonal property is lost and thus the performance degrades. Further the generation of OCV requires the knowledge about the chaotic vectors used by all other users which leads to additional processing of data. Therefore, OCV based non-coherent MA chaotic communication system finds advantage under synchronous transmission (for example, Broadcast transmission). For asynchronous transmission non-OCV with CSE-MMSE receiver can be used.

5.6 Summary

Non-Coherent multi user chaotic communication system using orthogonal chaotic vectors with Adaptive multi user receivers

- ✓ BER performance of non-coherent multi user chaotic communication system using OCV with adaptive multi user receivers are simulated and analyzed over AWGN channel for synchronous transmission.
- ✓ For non-coherent multi user chaotic communication system with CSE/IA based receivers the analytical expressions are derived and through simulation the accuracy of equations has been verified
- ✓ For non-coherent multi user chaotic communication system with LMS/NLMS/RLS based receivers the analytical expressions are derived by ignoring the estimation error. Therefore the equations obtained will serve as a lower bound for non-coherent multi user chaotic communication system.
- ✓ Through simulation and analytical expressions it is discovered that the performance of the system is better when Orthogonal Chaotic Vectors are used than compared to its counterpart using Non Orthogonal Chaotic Vector based.

6. Conclusions and Future Work

6.1 Conclusions

Multi user chaotic communication system using Orthogonal Chaotic Vectors with coherent receiver

- ✓ Multi user chaotic communication system using Orthogonal Chaotic Vectors with coherent receiver is simulated and analyzed over AWGN and multi path fading channels. Through simulation it is discovered that the performance of the multi user chaotic communication system using OCV will have better BER performance.
- ✓ It is very well known that using orthogonal vectors for multi user communication with perfectly synchronized receiver will have better performance due to the elimination of MAI. Thus, the results obtained for multi user chaotic communication using OCV with coherent receivers can be used as a benchmark results to compare other multi user chaotic communication system.

Non-Coherent multi user chaotic communication system using Orthogonal chaotic vectors with Adaptive multi user receivers

- ✓ BER performance of non-Coherent Multi user chaotic communication system using OCV with adaptive multi user receivers are simulated and analyzed over AWGN channel for synchronous transmission.
- ✓ For non-coherent multi user chaotic communication system with CSE/IA based receivers the analytical expressions are derived and through simulation the accuracy of equations has been verified
- ✓ For non-coherent multi user chaotic communication system with LMS/NLMS based receivers the analytical expressions are derived by ignoring the estimation error. Therefore the equations obtained will serve as a lower bound.
- ✓ Through Simulation and analytical expressions it is discovered that the performance of the system is better when Orthogonal Chaotic Vectors are used than compared to its counterpart using Non Orthogonal Chaotic Vector based.

6.2 Future Work

- ✓ Simulation and Analysis of coherent and non-coherent multi user chaotic communication system using orthogonal chaotic vectors receivers under asynchronous transmission condition.
- ✓ Complexity analysis of orthogonal chaotic vectors with respect to the number of bits or digits in precision.
- ✓ Complete parametric analysis of LMS/NLMS/RLS based adaptive multi user receivers used for non-coherent multi user chaotic communication.
- ✓ Simulation and analysis of non-coherent multi user chaotic communication using orthogonal chaotic vectors with adaptive multi user receivers over different types of multi path fading channels.

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